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A Modern Theory of Kuznets' Hypothesis

I. *Introduction*

The Kuznets hypothesis The character of evolution of the distribution of income along an economy's development process has been a theme with a long history in economic enquiry. The literature starts with the classic contribution of Simon Kuznets (1955), who was the first to identify economic growth as a determinant cause of long term changes in the distribution of income. Establishing his proposition on data from the time of industrialization of currently advanced nations, Kuznets (1955) initiated the idea that the inequality characterizing income distribution exhibits a non-monotonic trend along the process of economic development: it appears to widen during a society's transition from a pre-industrial to an industrial system, it remains stable for a while and narrows as more mature stages of growth are reached.[1] This systematic evolution of income distribution along a country's development path became known as the *Kuznets Curve* –an inverted *U*-shape relationship between income per capita and personal income inequality.

In his article, Kuznets lays out a simple model that places weight on the process of industrialization in driving the observed trends in the distribution of income. All developing countries are characterized by the coexistence of a traditional agricultural, and an industrial sector. The former is distinguished by its lower per capita income, and possibly narrower, but never wider inequality of distribution. Economic development proceeds by the rapid growth of industry, and the accompanied resource flows from agriculture. In earlier stages of this process, pronounced urban income inequalities exacerbate the countrywide magnitude of income variation. However, the rise over time in the relative weight of the industrial sector leads eventually to a narrowing of the overall inequality of distribution. A variety of forces interact to bolster the economic position of poorer segments of the population. As economic development proceeds, continually more individuals move from rural to urban areas, thus taking advantage of the opportunities of the relatively rich industrial sector. Furthermore, many workers who started out at the bottom rungs of the industrial sector walk up economically

[1] Kuznets (1955) formulates his proposition using available data from the industrialization period for the United States, England and Germany.

and socially. At the same time, a smaller size of the labor force is connected to agriculture, and this causes the relative wage rate in the rural sector to increase. These along with other, political and social considerations suggest a rise in the relative shares of lower-income groups.

The inverted-U: Evidence The subsequent literature evolved mainly in the direction of examining the robustness of the Kuznets curve on an empirical ground. Ideally, the evolution of inequality along the course of development should be examined in the historical context of individual countries. However, reliable time series data are scarce for most countries as we go back into the past. Consequently, the route has been to draw on cross country experience. Evidence on variations in inequality of countries that are at different stages along the development process provides information for exactly what is lacked for a single country. A bulk of cross-sectional studies has provided justification of the inverted-*U* hypothesis, leading to its acceptance in the 1970s as a stylized fact. This literature is represented by Paukert (1973), Ahluwalia (1976*a*,*b*), Adelman and Morris (1973), Chenery *et al*. (1974), Bacha (1977, 1979), Ahluwalia, Carter and Chenery (1979), and Adelman and Robinson (1989).

However, the alleged status of the Kuznets hypothesis was called into question by an array of subsequent studies. Papanek and Kyn (1986) challenged its empirical validity in an analysis of cross-section and time series data for 83 countries. They found that the support for the Kuznets relation is not strong, and may be weakening over time. In addition they point that there is considerable variability in income distribution at all levels of income, which is failed to be explained by the Kuznets effect. In a similar vein, Bourguignon and Morrison (1990) find a weak link between per capita income and income distribution in a cross-section study of developing countries. Anand and Kanbur (1993) have also suggested that the relation had weakened over time. Li, Squire and Zou (1998) suggest that the Kuznets hypothesis is generally in accord with cross-sectional observations obtained at a point in time. However, they present evidence that counter the validity of an inverted-*U* pattern over the course of evolution of individual economies. Their position is that the inequality of income distribution has remained relatively stable in the second half of the 20th century in a sample of 49 developed and developing countries. In a more recent contribution to the literature, Barro (2000) has re-established the inverted-*U* as a central theory in linking inequality to economic growth. In a panel study of closely 100 countries and covering from 1960 to 1995 the Kuznets hypothesis is established as a strong empirical regularity.

The 'trickle-down' theory Several theorists have concentrated more recently on extending the theoretical basis behind the Kuznets hypothesis. The proposed theories relate each in a different way to the notion of *trickle-down*. The idea is that with enough growth and little intervention to correct income inequality, the fruits of economic development will eventually filter or *trickle-down* to the poor, as the demands for what the latter can offer are magnified (Debraj Ray, 1998).

Greenwood and Jovanovic (1990) formulate a theory in which economic growth is inextricably linked to the development of financial markets and institutions. In their model intermediation structure is costly to build; hence, the level of financial development depends on the stage of the growth cycle. At the same time, a well-advanced financial system spurs economic growth by mitigating the effects of information and transaction costs, thus contributing to an efficient allocation of investment funds. Intermediaries provide savers with a distribution of returns on their investments that both is preferred and has a higher mean. However, investment through financial markets is costly, and relatively poor agents may not afford to use the superior technology. The theoretical validity of the Kuznets curve is rooted in the advancement of an economy's financial system, and the extent to which its services are spread across population. In its earlier stages, the process of economic growth is accompanied by the progressive development of financial intermediation. Since relatively rich individuals may only be able to take advantage of the developing financial markets, the variation of income initially widens. Along the course of development, the sustained improvement in the economic position of progressively more and more individuals translates into a distribution of higher initial endowments of capital. The economy approaches a state where the entire population may claim a share in the higher income prospects of the investment technology provided by the financial sector. Income disparities ultimately fade away as the benefits of development permeate more widely.

A closely related argument was developed by Aghion and Bolton (1997) in a model of endogenous income distribution that also generates the dynamics of the Kuznets curve. Individuals face two investment opportunities: a backyard activity that yields a deterministic rate of return, and an entrepreneurial technology with superior, yet uncertain revenue. The latter requires a minimum amount of capital investment, which agents may borrow in the capital market if endowed with sufficient initial wealth. In this model, it is the middle class that borrows to finance costly investment, whereas the very poor and rich agents act as lenders through their investment in the safe asset. The key feature that drives the relation between growth and wealth inequality is the endogenous determination of the cost of borrowing. In the early phases of development aggregate wealth, hence the supply of credit provided by the rich class of lenders, is small implying a high cost of capital. As capital is further accumulated, the wealth of rich lenders grows relatively faster, leading to widening wealth inequalities. However, as economic growth progresses, more and more funds become available to finance a progressively smaller pool of borrowers. The equilibrium lending terms shift in favor of borrowers, thus equalizing the distribution of wealth.

Another strand of literature emphasizes the role of technological progress in governing the pattern of income inequality. Studies within this field represent Galor and Tsiddon (1997*a*), Aghion and Howitt (1997), and Helpman (1997). Two technologies coexist, an old and a more advanced one, and individuals choose where they seek to be employed. Intergenerational mobility is represented by the choice of a different employment sector than that of one's parents. The model predicts that following periods of major inventions – the factory system, electrical power, computers– economies undergo a phase of rapid economic growth associated with enhanced intergenerational mobility and increased inequality. This outcome depends on characteristics of the technologically advanced sector, such as paying a higher marginal return to ability while a lower reward to the less able. Along the course of development, complex technologies gain accessibility to a wider range of individuals. This process has the effect of diminishing intergenerational mobility, hence reducing the inequality of income distribution.

In another study, Galor and Tsiddon (1997*b*) present a theory of trickle-down based on human capital accumulation and the expansion of technological knowledge that stems from it. The forces that drive economic growth in this setting are the accumulation of human capital and the advancement of technology with the former taking the leading role. Technological knowledge augments endogenously, and is the by-product of individuals' investment in enhancing own education. The vehicle through which technological progress contributes to growth is the accumulation of knowledge. The latter acts to enhance the marginal return to individual investment in education, thus feeding back in the accumulation of human capital. A key feature of the analysis is the postulate that the individual learning aptitude is determined, in addition to own investment of resources in education, by parental human capital and society's aggregate knowledge. The model yields the dynamics of an inverted-*U* path. An initially poor economy composed of an uneducated population is characterized by a highly equitable distribution of income. To its largest extent the entire population earns a fairly low income stemming from minimum skill and productivity levels. Investment in human capital is initially undertaken by individuals of high educational background, since they are the only ones with high effectiveness in own investment in education. The economy as a whole registers growth, but the benefits of this growth are highly concentrated in a relatively small number of individuals. Technical progress is likely to have a more uneven character at low to intermediate levels of income. As technological knowledge takes off, ultimately the gains find their way to everybody. And along with the growing educational status of the labor force, the economy enters a cycle of steadily declining inequality.

In this paper we seek to offer an additional contribution in the research elaborating on the theoretical underpinnings of the Kuznets hypothesis. Close in spirit to Galor and Tsiddon (1997*b*), our theory builds on the trickle-down hypothesis in a model where growth is driven by accumulation of human capital and the expansion of knowledge in society. Our attention is concentrated on the role of financial markets in determining the potential for acquiring education, and therefore the distribution of income earning capabilities. We explore what fundamental forces lead to the non-existence of credit institutions in the market for funding education, and show that the emergence of the latter may play a critical role in spurring economic development. As endogenous technological knowledge takes off, the externality effect arising from knowledge spillovers gives rise to inverted-*U* dynamics in the evolution of income distribution.

Human capital: A missing market We construct an overlapping-generations model in which private incentives induce agents to invest in education, and where non-rival inventions are the by-product of the education process. Pursuing to address the issue of income distribution we develop a model with heterogeneous agents distinguished on the level of innate learning aptitude. Individuals who belong in the same generation, and thus face the same social capital, are characterized by different human capital levels due to the postulated heterogeneity in the effectiveness of their investment in education. An individual's level of human capital upon entering the workforce determines her productivity of labor, hence her income at that period of life. The character of income distribution, and its evolution across time, is therefore governed by the distribution of human capital in society.

This paper may be viewed as contribution to the literature on the role of financial intermediation in determining the pace, and character of economic growth.[2] In line with the traditional view, we establish on theoretical ground that the development of financial markets constitutes an inextricable part of the process of economic development. In a model where the roots of development lie in human capital accumulation, our aim is to examine how intermediated credit may spur or hinder individual investment in education, hence economic growth. We assume that formal education is costly, in the sense that it incurs a direct pecuniary cost.[3] Individuals may not fund educational choices out of retained earnings, wealth or any form of inherited bequests.[4] Such investment must be financed from human capital loans through formal credit organizations.[5] In

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[2] The relationship between financial development and economic growth has long been examined in the macroeconomic literature. Early research on the topic is associated with the work of Goldsmith (1969), McKinnon (1973), and Shaw (1973). Important contributions more recently include Greenwood and Jovanovic (1990), Bencivenga and Smith (1991), King and Levine (1993), Saint-Paul (1992) and Levine (1992).

[3] We abstract from consumption or other form of expenditure incurred in the period the investment is made.

[4] The analysis in version VII looks at the case where individuals receive an endowment in the retirement age, independent of their prior income.

[5] Our emphasis is placed on the existence of credit support for tuition type expenses in education. The structure of the model implies that individuals do not demand credit to fund consumption, or other investment purposes. This channel of effect of intermediated credit on human capital accumulation has been investigated by De Gregorio and Kim (2000). In an economy with heterogeneous agents, financial institutions through the provision of *consumption* credit allow high-ability individuals to abstain from productive work in their youth, and devote the whole time endowment in education. On the other hand, agents with low efficiency in human capital investment may find it optimal to specialize in market activities, and use the financial markets to engage in intertemporal smoothing. By providing these opportunities for specialization, credit markets enhance the economy's average efficiency of education,

economies lacking such institutions, individuals are entirely barred from productive educational choices; a consequence of the failure of the credit market.

That credit markets for education loans may not function perfectly, or be entirely missing, is an argument with a long recognition in macroeconomics. Early on, Friedman (1962) attributed the source of the failure of this market to the intrinsic nature of human capital, in being embodied in those who possess it. It is thus impossible for the return of the investment to be passed on to lenders, or serve as collateral in the event of failure to repay. Moreover, it is particularly difficult to monitor the productive use of the loan, and the effort put up by the investor. The ability to make use of human capital may be unknown even to the borrower. Genuine bankruptcies and strategic default may well occur, with there being little that a lender can do to get his money back. These issues make the provision of credit in this market problematic.

Quite some research in the macroeconomic literature has adopted this idea; however most have formally modeled it on an *ad hoc* foundation, by imposing some form of exogenous borrowing constraints. Loury (1981) has examined the dynamics of income distribution in a stochastic model with an absent market for educational loans. Ljungqvist (1993) emphasizes the role of missing markets for human capital in explaining the persistence of underdevelopment in a world with free trade in consumption goods and physical capital. Buiter and Kletzer (1992) argue that the inability to borrow may reduce human capital accumulation in a model where individuals must self-finance own training costs. And Barro, Mankiw, and Sala-i-Martin (1995) re-examine the theory of convergence in income across countries in the context of a model in which financing for human capital investment is not available.

Endogenous debt constraints The focus in the present paper centers on the forces that lie behind the observed credit market imperfections in education funding, as well as the development of institutional infrastructure that may lead to overcome this market failure. Our contribution lies in integrating the theory of endogenous credit constraints into an analysis of the relationship between economic growth and the dynamic evolution of income distribution.

There are two alternative theoretical approaches within which debt constraints may emerge endogenously. The first builds on the premise that credit rationing arises as an optimal response of lending institutions to issues of asymmetric information. The core argument consists of the claim that moral hazard and adverse selection are interwoven in the lender-borrower relationship, and interfere with market behavior leading to a variety of failures in loan markets. There are several different microeconomic theories relating to private information problems that imply a form of credit rationing. These are mostly based upon the non-observability of labor input

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and consequently growth and welfare for all current and future generations. Evidence in support of the presence of this effect has been presented, for the United States and other OECD countries by Behrman *et al*. (1989) and De Gregorio (1996).

(moral hazard)[6], physical output[7], and individual ability (adverse selection)[8]. In the context of human capital analysis with external financing of education, Zeira (1991) shows that as a result of asymmetric information, credit may be endogenously rationed as a precautionary measure against the possibility of moral hazard. In the growth area, Tsiddon (1992) relies on moral hazard issues in the educational market to provide an explanation of the long run divergence of income levels across countries.

The second approach is based not on underlying information problems, but on the inability of creditors to enforce a loan contract. The central idea draws from the work of Schechtman and Escudero (1977), Eaton and Gersovitz (1981), and Manuelli (1986) in the international literature on sovereign debt. The framework was originally formalized in the area of partial insurance against idiosyncratic risk by Kehoe and Levine (1993). Their study marks a new tradition in modeling endogenous borrowing limits, and is the path taken in this study as well.[9] In their formulation, the system of creditors' legal rights allows the punishment of a borrower committing default in the form of her exclusion from future participation in formal financial markets. Defaulters are denied access to new loans in the credit market, and intermediate institutions have the legal right to seize the tangible assets in the debtor's possession. This renders lending through capital markets following default an irrational act. In this setting, *participation constraints* ensure that in equilibrium agents entering into a contract would at no time be better off contemplating default.

Outline of the essay A word about the essay's arrangement. The precise structure of the model is set out in the following section, in accompaniment of an elaborate discussion on how financial intermediation is embedded into the analysis. Section III presents an exposition of circumstances that may lead to a state of poverty persist over time, and the possibility of transition on a path of equilibrium unbounded growth. The dynamic impact of the process of development on the economy-wide income distribution is discussed in Section IV. The final two sections augment the core analysis each in a different direction. Section V allows for the economy's interest rate to be endogenously determined. In Section VI the analysis is extended to accommodate a higher, more general degree of heterogeneity. The analysis in Section VII is the exposition of a case where the effective punishment scheme imposes less stringent consequences on a borrower committing default. Although difficult and often very complex, the exposition offers complementary insight, which stands in need for delving deeper into the theory. A brief discussion in the end takes the role of final conclusion.

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[6] The reader may see Jaffee and Russell (1976), Stiglitz and Weiss (1981, 1987), Aghion and Bolton (1997) for more information.

[7] An interested reader may further look at Banerjee and Newman (1993), Galor and Zeira (1993).

[8] Jaffee and Stiglitz (1990) provide information on this subject.

[9] This enforcement mechanism has also been adopted by Azariadis and Lambertini (2003) in a pure exchange framework with overlapping generations. Andolfatto and Gervais (2006) has been the first study to embody a similar enforcement mechanism in a model with endogenous human capital formation, in which costly education is financed through private credit markets. De la Croix and Michel (2007) extend the latter analysis in a general equilibrium framework, allowing for the interest rate be endogenously determined.

II. *The model*

Demographic composition The model is a variant of the overlapping-generations model with production introduced by Diamond (1965) and Samuelson (1968). Time is measured as discrete intervals, beginning at time t=0. Individuals have finite life spans of three periods, with a new generation born in each period. We call an individual *young* in the first, *adult* in the second and *old* in the last period of life. An equal number of individuals enter and leave the economy in each period, implying a stationary population. Each new generation is composed of a continuum of individuals, with total measure normalized to unity, $i \in [0,1]$. Generations are named after their birth date.

Human capital accumulation and heterogeneity Human capital is defined to refer to the skills and knowledge level possessed by an individual. It is an intangible and inalienable factor that cannot be treated separately from those who create it or possess it. We assume that young agents are born with a minimum level of human capital $h_{\min} > 0$, which can be thought of as the ability to talk and coordinate with each other. While young, one may make an investment on individual improvement by devoting real resources to formal education. Education is costly in the sense that it incurs a direct pecuniary cost equal to q units of output per person.[10]

The individual's level of human capital upon entering the labor force depends on the effectiveness of her investment given that she engages in the education process. We postulate that the return on education is determined by the stock of knowledge in society while the investment is undertaken, and by the individual's innate ability.[11] Consistent with Tamura (1991) we postulate that the investment sector is characterized by an external spillover effect of human capital. The human capital of the average citizen contributes to enhance any individual's ability to acquire knowledge.[12] The assumption is further adopted that the magnitude of the external effect is strictly increasing in own talent.

In the words of Loury (1981), "…the term *ability* refers to all factors outside of the individual's control which affect his productive capacity". Allowing individuals be distinguished on the level of their innate aptitude provides a source of heterogeneity at a skill level.[13] With the exception of innate ability, hence their effective

[10] We abstract from other aspects of costly education, such as the sacrifice of leisure and the disutility from effort.

[11] Education may be considered as a form of vocational process, in which q can include the cost of tuition, books, tools as well as a subsistence level of consumption. Students learn from every adult who is currently alive. Thus, the level of human capital acquired by an individual who invests in education depends upon the average stock of human capital among all adults in the society, (H).

[12] The meaning attached to the concept of societal knowledge is that of being embodied in the human capital possessed by the members of the population. We do not make a conceptual distinction between the stock of disembodied knowledge, in other words knowledge in books, and that of being embodied in human inputs. In studies that do adopt this distinction (see Stokey, 1991, Laing *et al*., 2003) it is the potential of unbounded increases in the former that provides the basis for persistent growth. In our setting, endogenous never-ending growth is made feasible due to an intergenerational external spillover effect in the process of knowledge creation.

[13] The role of innate ability in shaping one's acquired human capital has been addressed in several studies in the literature. Levhari and Weiss (1974) use the term *uncertain inputs* to refer to innate talent as a determinant factor of earning capacity (at the completion of

learning parameter, individuals share access to a common non-linear investment technology. At the moment we assume that there are two types of agents in the economy, with *high* and *low* ability respectively. Since all individuals of a given type are identical, henceforth we characterize an agent by her type. The human capital level of an agent born in period *t* with ability A_j is represented by the following function[14]

$$h_{t+1}^{j} = \begin{cases} A_j^{\delta} H_t^{1-\delta} & \text{if invest} \\ h_{\min} & o.w. \end{cases}, \tag{1}$$

where *j* denotes an individual's type, $j \in \{L, H\}$, $\delta \in (0,1)$ and $H(t)$, $\forall t \geq 0$, represents the society's aggregate (and average) stock of human capital at date *t*.[15] Evidently, we assume that $A_L < A_H$, with $A_L > 1$[16] We further postulate the following condition

Assumption 1

$$A_L^{\delta} H_t^{1-\delta} > h_{\min}.$$

We assume that in every period agents with high- and low level of ability constitute fractions $\lambda_H = \lambda$ and $\lambda_L = 1-\lambda$ of the population, respectively. Then, in any given period *t*, the stock of the economy's aggregate (and average) level of human capital is

$$H_t = \lambda h_t^H + (1-\lambda) h_t^L \qquad \lambda \in [0,1], \tag{2}$$

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one's education). Individual ability is modeled through a random variable reflecting in part the unpredictable component of innate aptitude. The stochastic nature of the variable has the interpretation that, at the time when making a choice about the investment in her education, an individual has imperfect knowledge of exogenous characteristics such as her actual ability. A similar type of uncertainty has also been accommodated in other studies such as Eaton and Rosen (1980), Loury (1981), Snow and Warren (1990), and Benabou (1996, 2002). In our model we allow individuals to have perfect knowledge of their own aptitude; thus we abstract from the stochastic aspect of the latter variable.

[14] The literature builds on two alternative ways of production of intangible human capital. In the standard formulation proposed by Lucas (1988) human capital is produced within the household, and is determined solely by the time-investment in education. According to this approach, the price of new human capital is the implicit price evaluated by the household's utility. The other formulation has been employed by King, Plosser, and Rebelo (1988) and Rebelo (1991) and assumes that there is a market for new human capital. Human capital is produced in the education-service industry, and physical, in addition to human capital may serve as an input. The price of new human capital is the market price of education [Mino, 1996]. In the present context, our objective is to investigate the role of credit markets in the growth process of human capital, hence the latter formulation is more appropriate to adopt. However, since the relevance of the factor intensity condition is limited, physical capital is not incorporated as an input in the technology of the investment sector.

[15] It would be reasonable to postulate that an individual's level of human capital is a function of parental educational background. The role of quality of the home environment in human capital formation has been investigated theoretically, and empirically, by several authors (see Coleman *et al*. 1966, Becker and Tomes 1986, Bénabou 1996, Galor and Tsiddon 1997*a*, *b*). We abstract from such intergenerational linkage in human capital levels, so as to focus on technological spillovers across individual investors.

[16] That the low-level of ability must exceed unity is logically derived in footnote 18.

where we assume that in period $t = 0$ there exists an initial old generation with $H_0 > 0$. Without loss of generality we assume that $H_0 = h_{\min}$.[17] We note that $\lambda h_t^H \equiv H_t^H$ represents the human capital level of the group of high-ability agents in period *t* (this constitutes the high-ability fraction of individuals of generation *t*-1). Similarly, $(1-\lambda) h_t^L \equiv H_t^L$.

Equations (1) and (2) reveal that the investment sector is characterized by an (intergenerational) external spillover effect. Private human capital investment causes growth in the average stock of human capital, which increases the effectiveness of investment in education by later cohorts. Since individuals are finitely lived, the external effect is the only source of steady-state growth.[18] Growth can be sustained by continuing accumulation of the input that generates the positive externality. Since no individual decisions affect in an appreciable way the average skill level, no one takes this effect into account when deciding whether to invest in education.

Structure of individual's life We lay out the decision-making process of a representative member of generation *t*. As mentioned, in the first period of life a choice must be made whether to enter the educational sector, and acquire human capital in excess of $h_{\min}$. If the individual decides to invest in education she must incur the cost *q*. In the absence of any initial wealth or labor income, the cost of education must be financed by borrowing in the credit market.

Denote first period's saving (borrowing) by $s_{1t} > 0$ (< 0). Assuming that this period's consumption is not valued, the budget constraint is expressed by

$$s_{1t}^j = \begin{cases} -q & \textit{if invest} \\ 0 & \textit{o.w.} \end{cases} \quad \forall j \in \{L, H\}, \tag{3}$$

in other words, the young borrows.

In the second period of life adults enter the labor market, supplying one unit of time inelastically.[19] We normalize units so that output produced is equal to the human capital employed. That is, the labor income of an individual with human capital h_{t+1}^j is given by

$$y_{t+1}^j = h_{t+1}^j, \tag{4}$$

[17] The assumptions $A_L\ H_t > h_{\min}$ and $H_0 = h_{\min}$ combine to imply that $A_L > 1$.

[18] This externality may be distinguished from the conventional transmission of human capital within households in the literature (e.g. Becker and Tomes, 1986). In an overlapping generations model it has been often interpreted as intergenerational externality (e.g. Stokey, 1991; Bovenberg and van Ewijk, 1997; Hendricks, 1999). We do not assume externality in output production (e.g. Lucas, 1990), human capital production (e.g. Azariadis and Drazen, 1990) or training time among individuals within a generation (e.g. Chamley 1993; Benhabib and Perli 1994) in order to exclude the possible indeterminacy of equilibrium paths. Tamura (1990) suggests a similar spillover effect in the technology of the educational sector.

[19] We assume that workers do not acquire human capital through on-the-job training.

where y_{t+1}^{j} stands for the individual wage income earned in period t+1. The hypothesis that income earning ability depends upon innate aptitude is consistent with evidence from the empirical literature. Griliches and Mason (1972) provide some direct evidence about the positive role of ability. The vast literature about returns to human capital supplies some indirect evidence, provided that education is positively correlated with ability [Galor & Tsiddon, 1997*a* p.365].

Let c_{2t+1}^{j} and s_{2t+1}^{j} denote respectively the second-period consumption and saving of a member of generation t with ability A_j, $j = L, H$. Earned income net of debt repayment is allocated between consumption and savings[20]

$$c_{2t+1}^{j} + s_{2t+1}^{j} = y_{t+1}^{j} + R\, s_{1t}^{j}. \qquad (5)$$

Using (4) the budget constraint in the second period of life is expressed as

$$c_{2t+1}^{j} = h_{t+1}^{j} + R\, s_{1t}^{j} - s_{2t+1}^{j}. \qquad (5')$$

And using (1) and (3), (5') is written as

$$c_{2t+1}^{j} = \begin{cases} A_j^{\delta}\, H_t^{1-\delta} - R\, q - s_{2t+1}^{j} & \text{if } q > 0 \\ h_{\min} - s_{2t+1}^{j} & \text{if } q = 0 \end{cases}. \qquad (5'')$$

In the third period of life agents retire, using the entire return of savings for consumption

$$c_{3t+2}^{j} = R\, s_{2t+1}^{j}, \qquad (6)$$

where c_{3t+2}^{j} denotes consumption in old age of a member of generation t who is of type j, $j \in \{L, H\}$.[21]

Individual's optimization problem All young agents share identical preferences, defined over consumption in the second and third period of their lives. The preferences of an individual of type j born at time t are represented by the intertemporally additive utility function

$$U_t^{j} = \beta \ln\left(c_{2t+1}^{j}\right) + (1-\beta)\ln\left(c_{3t+2}^{j}\right), \qquad (7)$$

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[20] We abstract from other forms of transferring consumption from one period to another, such as fiat money or types of storage technology. Income may be transferred from the second to third period only by lending through the financial system.

[21] Alternatively, one may assume that individuals receive in old age an endowment, ω_3, irrespective of their educational status. This may be thought of as a type of retirement income. The analysis of this case is demonstrated in Section VII.

where U_t^j stands for the lifetime utility of a member of generation *t*, who is of type *j*. In the first period of life an agent decides whether to acquire education, a decision determining her gross lifetime income. At the same time, the individual shall decide whether to default on her debt or remain committed to her obligation. The joint decision determines the agent's *net* lifetime income, which she allocates between second- and third-period consumption so as to maximize her lifetime utility (equation 7) subject to the two budget constraints (equations 5" and 6). There exists a unique and interior solution to the optimization problem that is expressed by

$$s_{2t+1}^{j*} = \begin{cases} (1-\beta)\left(A_j^{\delta} H_t^{1-\delta} - Rq\right) & if \;\; q>0 \\ (1-\beta)h_{\min} & if \;\; q=0 \end{cases} \quad \forall j = L, H\,. \tag{8}$$

Setting equation (8) into (5") we obtain the optimal level of second-period consumption

$$c_{2t+1}^{j*} = \begin{cases} \beta\left(A_j^{\delta} H_t^{1-\delta} - Rq\right) & if \;\; q>0 \\ \beta h_{\min} & if \;\; q=0 \end{cases} \quad \forall j \in \{L, H\}. \tag{9}$$

Similarly, substitution of (8) into (6) yields the optimal level of third-period consumption, expressed by

$$c_{3t+2}^{j*} = \begin{cases} (1-\beta)R\left(A_j^{\delta} H_t^{1-\delta} - Rq\right) & if \;\; q>0 \\ (1-\beta)R\,h_{\min} & if \;\; q=0 \end{cases} \quad \forall j \in \{L, H\}. \tag{10}$$

Financial intermediation and contract enforcement We assume that there exists a market of financial institutions that allow individuals to trade in financial markets, as well as to obtain credit for human capital investment. Financial institutions intermediate economic activity between (adult) individuals who save to enhance third period consumption, and those who borrow. We postulate the existence of $\kappa = 1,...,K$ members within the financial system. As referred by Ljungqvist and Sargent (2004), and first espoused by Green (1987), we suppose that financial institutions have access to a capital market "outside" the economy, where they can borrow or lend at a riskless real interest rate. Individual households do not have access to this outside market, and they are prohibited from borrowing or lending with each other (Ljungqvist and Sargent, 2004). Should they engage in intertemporal trade, they must do so solely through the intermediary system. At the moment we assume that private contracts may be signed at a fixed (gross) real interest rate, *R*, charged on both deposits and borrowing.

We assume that young individuals, once obtaining credit they always invest in human capital. By assumption, an educational loan is not to be used in alternative ways. When adult, educated individuals have the option of going bankrupt, thus evading existing debt payments. A creditor cannot ensure that the borrower will meet her obligations.[22] In the terminology of Jaffee and Russell (1976), individuals in our model are *potentially dishonest* in the sense that when there are incentives to default they always choose to do so.[23] We shall consider that this lack of commitment in honoring a private contract is one-sided: deposit institutions by supposition always honor their promises to future payment streams. In an environment with this asymmetry in credible commitment, contracts must be *self-enforcing* in the sense that households are induced by their own self-interest to repay their creditors (Ljungqvist and Sargent, 2004).[24]

Started by the international literature on sovereign debt, invoking a punishment scheme on defaulting borrowers has seemed to be the only way out of the particular difficulties engendered by one- or two-sided lack of commitment on one's promises. In line with Kehoe and Levine (1993), the prospect of complete and permanent exclusion from the financial market is the credible threat that provides in our setting the motive for individual commitment to contract obligations. The punishment strategy following an act of default is twofold: on one hand, individuals have no access on financial assets as medium of saving. Creditors' legal rights allow them to seize the entire future savings in the debtor's possession implying that it is in the strongest interest of the latter to carry no savings in the formal financial sector. On the second hand, defaulters are denied access to new loans in the credit market. Given the structure of our model, enforcement may not be supported by long-term cutoff from further credit. It is never optimal for financial institutions to provide credit to adult individuals, since no debt is honored in the last period of life (Kehoe and Levine, 1993). So, were they to default no payment could be imposed on them. This reads into the constraint

$$s_{2t+1}^{j*} \geq 0 \quad \forall j = L, H , \tag{11}$$

which constitutes an *individual rationality constraint* for members of the banking system. Hence, the only individuals who may be able to borrow are the young who choose to enter the educational sector. Consequently, in our context the Kehoe and Levine enforcement mechanism reduces into the prohibition of placing savings within the formal financial system. The only cost of default is the loss of the ability to smooth consumption along the course of one's life.[25]

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[22] The lender could ensure repayment of debt if workers were, for example, required to disclose information to become employed and reveal information while employed (*see* Fender and Wang 2003).

[23] Jaffee and Russell (1976) distinguish between two types of individuals, the honest, who are *pathologically* honest since they refuse to default even when there is an incentive to do so, and the dishonest that are *potentially* dishonest, since there are cases where they reveal only honest behavior [p.652].

[24] Recall that limited contractual enforcement in our setting is due to the inalienability of human capital, which cannot be seized and transferred to a creditor in the event of default. Lack of enforcement is strengthened by the assumption that existent individual endowments in old age consist of no collateral goods.

[25] The fact that in our environment the full-exclusion scheme is reducible to a simpler form is not material for the mechanism's effectiveness in punishing a defaulting borrower. The vital component of the Kehoe and Levine scheme is that agents are deprived of the opportunity to invest their savings. It has been shown by Bulow and Rogoff (1989), and later been confirmed by more contemporary studies (see Bond and Krishnamurthy, 2004) that the sole elimination to a borrower on default status of his right to

The full-exclusion scheme of Kehoe and Levine (1993) is structured on the ground of two critical assumptions that concern the legal rights of financial institutions, and the interrelationships among them. The ability to prohibit a borrower who repudiates on her debt from actively participating in the formal financial market rests on the implicit existence of a legal entity (legislation and judicial system), capable to observe any such trade actions and confiscate the relevant net payments. In the Kehoe and Levine model the economy's legislative system grants creditors the power to appropriate assets in the debtors' possession, and preclude their access to future contingent claims markets.[26] Extensive legal power of lenders is a necessary prerequisite to support a decentralized allocation in a structure of partial commitment. We should remark that the existence of a competitive equilibrium is established in an environment that abstracts from issues of competition among financial institutions. Kehoe and Levine presuppose that members of the banking system form a stringent coalition, thus fully coordinating their decisions. [27] In like manner, we restrict our attention to the case of no competition within the financial market. For the sake of simplicity, we assume without loss that the borrower deals with a single bank, interpreted to represent the entity of financial sector.[28]

While we recognize the widely accepted status of the Kehoe and Levine arrangement in studies with contract design problems, we must make a remark on the criticism carried by Bond and Krishnamurthy (2004). The authors dealt thoroughly with the minimal practical value of the scheme, reasoning that the full-exclusion enforcement rule is hard to identify in observed institutions, or resemble laws governing borrowers' defaulting on debt and declaring bankruptcy [pp.691-2]. We argue that only an aspect of this criticism cherishes validity in our context, and that the Kehoe and Levine scheme is a good representation of the actual legal doctrine regulating educational loan markets. In connection with any type of non-educational credit, it is indeed the case that the *permanent* character of prohibitions imposed on delinquent borrowers is too stringent an assumption in keeping with a plausible theory. It is common to most if not all legislative systems that an individual is entitled to declaring bankruptcy on her debt, thus claiming an opportunity to a "fresh start". However, the ability to have

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further credit, has no impact on his incentive to remain loyal at first place. If individuals have access to a savings market irrespective of their contract commitment, the optimal loan size for banks is zero. We ought to make the remark that this proposition is guaranteed only insofar specific conditions are assumed.

[26] From the standpoint of the authors, individuals may not be barred from trade in spot markets, nor can their endowments be taxed due to unfulfilled obligations on private contracts. The view is justified on the basis that trades in spot markets are anonymous, and that there may not be a physical separation of private endowments from individual owners. At the same time, agents must identify themselves to make contracts and to collect on them in contingent claims markets. Therefore, creditors may seize the assets of a debtor who defaults on her debt, and may keep track of any future attempts of hers to enter contingent claims markets. As a consequence, they can exclude the borrower from engaging in intertemporal transactions, while tax her individual assets [Kehoe and Levine, 1993 p.869].

[27] The authors say nothing about how this coordination actually occurs. Formal consideration of the competition within the financial sector has been carried by Bond and Krishnamurthy (2004), in an analysis which in the main contributes a critic on the Kehoe and Levine absolute-exclusion scheme.

[28] The conclusions of the model are the same whether one assumes that the borrower deals with a representative bank, or whether the entire banking sector behaves as a coalition (monopolist). Bond and Krishnamurthy (2004) present an elaborate study on salient features of the Kehoe and Levine scheme.

one's debt waived does not extend in the area of educational loans. An individual debtor continues to be liable for all obligations on student loans even after her claim to bankruptcy is successfully pursued.[29] These debts are legally discharged only in the event of full repayment. The creditors' legal right to confiscate the private assets of the debtor being bound by obligation establishes the empirical validity of the full-exclusion scheme in the area of educational investment.

The second aspect of criticism concerns the Arrow-Debreu trading arrangement of complete markets. In the Kehoe and Levine structure competitive markets meet at date 0 to trade claims to consumption at all times $t > 0$, that are contingent on all possible realizations of events up to t. In this respect, Bond and Krishnamurthy (2004) posit that the implementation of debt constraints is in fact complex state- and date-contingent specifications of payments [p.691]. The crux of the criticism is that the computation of this system of payments being highly information intensive amounts to an insuperable task.[30] In the words of Ljungqvist and Sargent (2004) "…we are assigning a very demanding task to the *invisible hand* who must not only look for market-clearing prices but also check participation constraints for all agents and all states of the world" [p.740]. The acceptance of this criticism clearly does not lend power to the practical value of our theoretical construct.

III. *Equilibrium*

Poverty trap We have claimed that conditional on default, an individual may be barred forever from asset trading in financial markets. As the preceding analysis brings out, the character of such prohibition is defined by the law surrounding the creditors' right to loan repayment. When legislation supports absolute and permanent consequences upon default, as in the Kehoe and Levine scheme, we say that lenders come out with a strong legal position (or else *strong legal rights*). The legislative system entitles them to full loan repayment in every circumstance; a situation we stated describes educational credit in well-advanced financial systems. In ever weaker positions, full loan repayment may not be enforced; a situation we here term *weak legal rights*. Practically this is true when debt for higher education may be discharged after declaring bankruptcy, and when unsecured educational loans gain low priority in the event of property liquidation. As is generally supposed, the power to which creditors are potentially entitled has a critical effect on their readiness to finance investment (La Porta, *et al*, 1998). We postulate that when the legal system offers little or no protection to lenders, failing on loan repayment is in essence accompanied by no penalty. As a consequence, strategic default may well be

[29] The US Bankruptcy Code does not exempt individuals from their obligation on educational debt. The legal doctrine was formalized in the United States in 1978. Subsequent amendments of legislation (the last passed on in 2005) extended the array of educational loans covering all credit aiming to higher education, funded by governmental or private source. Similarly, in the United Kingdom a debtor remains bound by her obligations on government educational loans after the claim to bankruptcy.

[30] Reciting Bond and Krishnamurthy (2004) "… the computation procedure requires the central (judicial) authority to possess knowledge on agents' production and consumption possibilities; knowledge that is unknown how it could be obtained in a decentralized competitive environment." [p.691].

expected, making credit institutions least eager to engage in loan financing. The optimal response is to deny the provision of any loan, resulting in an extreme form of credit rationing; an actual feature of educational credit markets in most economies of the world.

Weak legal protection on the account of creditors impedes the development of a private educational loan market leading to a competitive equilibrium characterized by no investment in higher education. In a setting where growth hinges on the accumulation of human capital economic development must then come to a halt. This is self-evident in a system where education may not be otherwise socially provided, an assumption that has been postulated at the outset of our analysis.

Given that a member of generation t receives education her intertemporal consumption if she chooses to default is described by the following equations

$$\left(c_{2t+1}^{j*}\right)^{WR,D} = \beta A_j^{\delta} H_t^{1-\delta} \qquad q > 0, \forall j \in \{L, H\}, \forall t \geq 0, \tag{12}$$

and

$$\left(c_{3t+2}^{j*}\right)^{WR,D} = (1-\beta) R A_j^{\delta} H_t^{1-\delta} \qquad q > 0, \forall j \in \{L, H\}, \forall t \geq 0, \tag{13}$$

where WR (SR) refers to a system of weak (strong) legal rights on creditors' account, and D (ND) stands for default (no-default) respectively. When remaining loyal to contract commitment optimal adult- and old-age consumption is given by, respectively

$$\left(c_{2t+1}^{j*}\right)^{ND} = \beta\left(A_j H_t - Rq\right) \qquad q > 0, \forall j \in \{L, H\}, \forall t \geq 0, \tag{14}$$

and

$$\left(c_{3t+2}^{j*}\right)^{ND} = (1-\beta) R \left(A_j H_t - Rq\right) \qquad q > 0, \forall j \in \{L, H\}, \forall t \geq 0. \tag{15}$$

where $\left(c_{\nu t+1}^{j*}\right)^{WR,ND} = \left(c_{\nu t+1}^{j*}\right)^{SR,ND} \equiv \left(c_{\nu t+1}^{j*}\right)^{ND}$ for $\nu = 2, 3$.

Evidently, utility is higher when evading debt obligations due to higher second-period consumption, and because agents can still engage in intertemporal smoothing through saving. Hence,

$$V_t^{WR,D}(j) > V_t^{WR,ND}(j) \qquad q > 0, \forall j \in \{L, H\}, \forall t \geq 0, \tag{16}$$

where

$$V_t^{WR,D}(j) = \ln\left\{\gamma A_j^{\delta} H_t^{1-\delta}\right\}, \qquad q > 0, \forall j \in \{L, H\}, \forall t \geq 0, \tag{17}$$

and

$$V_t^{E,ND}(j) = \ln\left\{\gamma A_j^{\delta} H_t^{1-\delta} - Rq\right\} \quad q > 0, \forall j \in \{L, H\}, \forall t \geq 0, \tag{18}$$

with $V_t(\cdot)$ standing for the indirect utility function of a member of cohort *t*, while *E* denoting that the individual has received education when young. It obviously applies that $V_t^{WR,ND}(j) = V_t^{SR,ND}(j) \equiv V_t^{E,ND}(j) \quad \forall\, j = L, H$, while $\gamma \equiv \beta^{\beta}(1-\beta)^{1-\beta} R^{1-\beta}$. We come to the conclusion that an individual who acquires education shall always commit default on her debt.

Were one to receive no education she would earn the unskilled income $h_{\min}$, hence the lifetime utility

$$V_t^{NE}(j) \equiv V_t^{NE} = \ln\{\gamma h_{\min}\} \quad \forall j \in \{L, H\}. \tag{19}$$

Drawing on *Assumption* 1 we infer that remaining unskilled is never the preferred choice. It is evident that

$$V_t^{WR,D}(j) > V_t^{NE} \qquad \forall j \in \{L, H\}, \forall t \geq 0. \tag{20}$$

Obtaining credit, although desirable, in no circumstance is feasible. Due to the certainty of default behavior it is individually rational for members of the financial system to deny the provision of any loan.

Owing to the rationing of all credit, the total measure of population remains uneducated earning the minimum income of unskilled labor. On the assumption that a system of weak legal rights prevails in each period of the time interval $t \in [0, r-1]$ $r > 0$, the equilibrium path has the characteristics

$$h_{t+1}^{j} = h_{\min} \quad \forall j \in \{L, H\}, \forall t \in [0, r-1], \tag{21}$$

and

$$y_{t+1}^{j} = h_{\min} \qquad \forall j \in \{L, H\}, \forall t \in [0, r-1]. \tag{22}$$

The competitive outcome along this equilibrium path prescribes that the economy produces the time-invariant quantity

$$Y_{t+1}^{NE} = H_{t+1}^{NE} = h_{\min}, \tag{23}$$

where at the outset we postulated $y_0^j \equiv y_0 = h_{\min}, \forall j = L, H$, thus $Y_0 = h_{\min}$. The economy here is void of any growth, with production merely contributing to sustain the starting level of effective labor, and output.

Potential for growth: A fraction of population invests We proceed with defining equilibrium paths along which the potential for ever sustained growth is realized within the structure of this model. We consider two such equilibria, so constructed as to give an explicit proof of the *trickle-down* benefits of economic growth. As our basis, an equilibrium is taken to exist in which a particular group of individuals, those with the highest investment return, can only choose to engage in education. Owing to the accumulation of human capital and the associated externality on future generations' productivity, the economy reaches the state where the aggregate of all agents invest in individual improvement. The financial sector, eager to support educational decisions of all and any prospects, carries the economy on a new dynamic path on its way to development. With reference to existence issues, we shall stress that the critical requirement for perpetual growth is the provision of strong legal protection for creditors, should contract obligations be violated. Even though growth is an endogenous outcome in our model, its manifestation ultimately hinges on the ability of lenders to enforce loan repayment by imposing financial consequences on those who default. We take as our basis that effective on period $t = r$, $r > 1$, legislation entitles creditors to claim full loan repayment from delinquent borrowers (case of *strong legal rights*). Lenders may seize the entire assets of a debtor in default, effectively prohibiting the latter from any act of saving as time unfolds.

We introduce a formal definition of the postulated heterogeneity at an individual income level. We recall that individuals are distinguished on the level of their innate aptitude, and consequently their rate of return from education. Our supposition is that there exist two types of agents in the economy, those with *high* and *low ability* respectively. This is chosen to be our benchmark case, which we subsequently augment to allow for a (countable) infinite number of types. We employ the definition:

Definition 1 An agent born in time period *t* is said to be of *high type* if her (gross) return from education can support her honoring of debt obligations. Accordingly, *low type* agents are discerned by an earned income as low as not in the least covering loan repayment. This definition reads into the postulates

Assumption 2

$$A_H^{\delta} H_t^{1-\delta} > Rq \qquad \forall t \geq 0. \tag{α}$$

$$A_L^{\delta} H_t^{1-\delta} < Rq \qquad \forall t \geq 0. \tag{β}$$

The meaning ascribed to the employed distinction thus has to do with the feasibility in carrying out one's contract commitments. Assumption (2β) is read to mean that investment in human capital does not pay off if one is to remain committed to her debt liability. Were a low-type agent to obtain credit, she would always default on her obligations, however honest in intention.

We remark here that while an individual knows her type when deciding whether to invest in education, innate ability has nevertheless an unobservable quality. The private information of one's own type is not to be publicly revealed, or otherwise obtained by a credit institution. The fact that individuals cannot be identified on the basis of their expected rate of return on education brings about the constraint that borrowers choose to conform to contract arrangements in equilibrium.[31] Such incentive is assuredly instilled by the stringent nature of consequences of our punishment scheme. With innate ability being non identifiable, the only possibility to deviate from an equilibrium outcome with rationing to everyone who applies for credit is with *self-selection* of individuals to different choices. Carrying the analysis in formalized language we introduce the following definition:

Definition 2 A contract is said to be *self-enforcing* if it is individually optimal for borrowers to conform to prior arrangements at every date and contingency.[32]

It occurs that within our context, the contract design with the aforementioned absolute consequences following default elicits only promise-keeping behavior. The present value of (indirect) utility associated with the consumption stream after repudiating on one's debt is given by

$$V_t^{SR,D}(j) = \beta \ln\left\{A_j^{\delta} H_t^{1-\delta}\right\} + (1-\beta)\ln\{0\} \to -\infty \qquad \forall j = L, H . \tag{24}$$

There results, consequently, that

$$V_t^{E,ND}(j) > V_t^{SR,D}(j) \qquad for\ j = H , \tag{25}$$

where the lifetime (indirect) utility when adhering to the contract is given by (18).

The high ability agents choose to obtain education and repay their debt for the reason that default is too costly. Condition (25) cannot possibly hold for low-type agents since the logarithmic function $V^{E,ND}(j)$ is non-definable on a negative argument (*low* income realization does not enable debt repayment). Insofar as the only possibility is to renege on the agreement, the household attains the lowest utility level associated with no consumption smoothing. $V^{E,ND}(L)$ in effect degenerates to the lowest value of individual welfare $V^{SR,D}(j) \to -\infty,\ \forall j = L, H$. Consequently, we quote the proposition:

Proposition 1 The contract arrangement (q, R) offered in a system where borrowers have no access to savings opportunities conditional on default is a self-enforcing contract. Given the feasibility of loan repayment, it is never optimal to renege on agreed obligations.

Further, we may prove without difficulty

Proposition 2 A private credit market for human capital investment is *sustainable* if and only if

[31] It is to be noted that the value of innate ability for each type, hence educational productivities are known to moneylenders. The latter have knowledge of the feasibility constraints, as expressed by *Assumption* 2α, β. However, potential borrowers may not be discerned on the basis of their individual type. Innate aptitude is non-identifiable, thus rendering the explicit rationing of low-type agents impossible.

[32] The definition is borrowed from Ljungqvist and Sargent (2004, p.640).

- Given the feasibility of loan repayment agents seeking credit are offered a self-enforcing contract.
- Individuals for whom debt repayment is non-feasible prefer to receive no education.

Proof

We have established that high-ability agents, once obtaining credit always adhere to the contract agreement. The second necessary condition requires that the low ability agents choose (optimally) to remain unskilled. For individuals of this type, investment in human capital comes at the cost of sacrificing the opportunity to smooth consumption. Therefore, it holds true that they prefer earning the low income of unskilled labor, while maintaining their ability to ensure their old age consumption via saving in tangible assets. Upon invoking that $V^{E,ND}(L) \to V^{SR,D}(L) \to -\infty$, and recalling expression (19) $V_t^{NE} = \ln\{\gamma h_{\min}\} > 0$, it is trivially proved that low-talented individuals indeed prefer to seek no education.[33] Within the present context on the basis of the postulate that last-period consumption consists exclusively of previous savings, the postulated definitions of agent types (*Assumption* 2), and with the use of the logarithmic utility function (7) we have formally arrived at the sentence that the credit educational market is privately sustainable.‖

We conclude with the following proposition

Proposition 3 A competitive equilibrium with a subset of the population acquiring privately financed education exists on the condition of occurrence of the following requirements

- A positive measure of individuals choose optimally to obtain education.
- The credit market is sustainable.
- Savings be non-negative for both *H* and *L* types.

Proof

- The first condition constitutes the *participation constraint* for the borrower side. The high-ability agent can always guarantee herself the present value of utility V^{NE} by supplying unskilled labor. The contract must offer her at least this utility level. Therefore, it must be satisfied that

$$V_t^{E,ND}(H) > V_t^{NE}, \tag{26}$$

which, drawing upon equations (18) and (19), applies if and only if

$$A_H^{\delta}\, H_t^{1-\delta} > Rq + h_{\min}. \tag{26'}$$

[33] The proof is obviously weakened as soon as we remove the assumption of last-period consumption being exhaustively determined of prior savings. This case is presented in section VII.

Expression (26') is also referred as the *individual rationality constraint* for the high-type agent (IR_H). To ensure the validity of *Assumption* 2α and of condition (26') it suffices to impose their intersection, which is expressed by the latter relationship. We are certain of the truth of (26') in all periods $t \geq r$ given that

$$H_r = h_{\min}, \tag{27}$$

and the constraint applying solely in period $t = r$

$$A_H^{\delta}\, h_{\min}^{1-\delta} > R\,q + h_{\min}. \tag{26''}$$

- Drawing on *Proposition* 2 we transfer the conclusion that the postulated definitions of agents types (*Assumption* 2) suffice as proof of the sentence that the educational credit market is privately sustainable.
- The last statement of *Proposition* 3 imposes the individual rationality constraints for the banking system, as expressed by relationships (11). Invoking equation (8) we obtain

$$s_{2t+1}^{H\,*} = (1-\beta)\left(A_H^{\delta}\, H_t^{1-\delta} - R\,q\right) > 0 \qquad \forall t \geq 0, \tag{28}$$

which is strictly positive on the basis of the definition of the high-type agent (*Assumption* 2α). With respect to the low-talented individuals saving is represented by

$$s_{2t+1}^{L\,*} = (1-\beta)\, h_{\min} \;>\; 0 \qquad \forall t \geq 0. \tag{29}$$

The proof of *Proposition* 3 consists basically of imposing *Assumption* 2β along with constraint (26''). The validation of the remaining relationships is inferred by means of logical reasoning.‖

The dynamic evolution of the society's stock of human capital along this equilibrium path is governed by the first order non-linear difference equation

$$H_{t+1} = (1-\lambda) h_{\min} + \lambda A_H^{\delta}\, H_t^{1-\delta} \qquad \forall t > r. \tag{30}$$

The solution to the linear difference equation $(i.e.\ \delta = 1)$ characterizes the current stock of knowledge as a function of society's historically given H_0, the level of human capital of unskilled labor, and the ability as well as the measure of those who invest. Having presupposed $H_0 = h_{\min}$, the solution is described by

$$H_t = \begin{cases} h_{\min}\left[\dfrac{(1-\lambda)+\lambda(\lambda\,A_H)^t(1-A_H)}{1-\lambda\,A_H}\right] & \text{if } \lambda A_H \neq 1 \\ [1+(1-\lambda)t]\, h_{\min}, & \text{if } \lambda A_H = 1 \end{cases} \qquad \forall\, t \geq 1. \tag{31}$$

It is easily observed that in the case of $\lambda A_H \neq 1$ the numerator of the term in brackets may most likely be negative. This calls to impose the relation $1-\lambda A_H < 0$, or equivalently $A_H > 1/\lambda$. Evidently, this is stricter compared to the initial assumption $A_H > 1$.

Sustained growth carried by the entire population The equilibrium we described involves a constant fraction of each generation (the measure of high-type agents) acquiring education, and earning income $A_H^{\delta} H_t^{1-\delta}$. The remaining population chooses to remain unskilled and earn the minimum income level $h_{\min}$. As long as the validity of *Assumption* 2*β* and constraint (26") is ensured, the composition of the labor force between educated and uneducated individuals is analogous to the fraction of each generation being genetically of high aptitude. The latter is established *a priori* to be a stationary variable across all time periods.

It has been an initial motivation to prove the existence of possibility that the educational status is affected dynamically as the economy evolves along the path of perpetual unbounded growth. Within the structure of this model, sustained growth carries the potential that the measure of population who find it optimal to invest in education changes endogenously. We prove that due to perpetual growth the stock of aggregate knowledge reaches a threshold level above which individuals of the low type as well choose optimally to invest in the acquisition of human capital. We establish specific generality of this result by running through the case of linear human capital technology, $(i.e.\ \delta = 1)$. We do not take the foregoing proof beyond the linear case due to the particular complexity in solving non-linear difference equations in abstract form.

The theorem states that

Proposition 4 There exists a time period $\tau > 0$, where $\tau \in [r+1, \infty)$, in which the income realization of educated low-type agents exceeds the threshold level that defines education the optimal choice. In other words,

$$A_L^{\delta} H_t^{1-\delta} \begin{cases} \geq Rq + h_{\min} & \forall t \in [\tau, \infty) \\ < Rq + h_{\min} & \forall t \in [0, \tau - 1] \end{cases}. \tag{32}$$

Considering the linear human capital technology, τ is defined as

$$\tau = \begin{cases} \dfrac{\ln[\Theta]}{\ln[\lambda A_H]}, & if\ \lambda A_H > 1 \\ \dfrac{Rq + h_{\min}(1-A_L)}{A_L h_{\min}(1-\lambda)}, & if\ \lambda A_H = 1 \end{cases}, \tag{33}$$

with $\Theta \equiv \frac{(Rq+h_{\min})(1-\lambda A_H)-A_L h_{\min}(1-\lambda)}{A_L h_{\min}\lambda(1-A_H)}$.

Proof

Relationship (32) written in linear form yields

$$A_L H_t \geq Rq + h_{\min} . \tag{34}$$

We use the solution of the aggregate human capital stock as given by (31), to substitute for H_t. Equation (33) is then derived in a straightforward manner.

We can prove the existence of time period τ only on the condition that the latter is greater to unity. More precisely, the theorem is true if and only if

- *Case* $\lambda A_H > 1$

$$\Theta > \lambda A_H , \tag{35}$$

which leads to a standard second-order polynomial

$$\left(\lambda^2 A_L h_{\min}\right)A_H^2 - \lambda\left[Rq + h_{\min}(1+\lambda A_L)\right]A_H + Rq + h_{\min}\left[1 - A_L(1-\lambda)\right] > 0 . \tag{35'}$$

The expression is positive insofar as, either

$$A_H < \frac{Rq + h_{\min}(1+\lambda A_L) - \sqrt{\Omega}}{2\lambda A_L h_{\min}} \equiv A_H^1 , \tag{36α}$$

or,

$$A_H > \frac{Rq + h_{\min}(1+\lambda A_L) + \sqrt{\Omega}}{2\lambda A_L h_{\min}} \equiv A_H^2 , \tag{36β}$$

where we note that $\Omega \equiv \left[Rq + h_{\min}(1+\lambda A_L)\right]^2 - 4A_L h_{\min}\left[Rq + h_{\min}(1 - A_L(1-\lambda))\right] \geq 0$, and $A_H^1 > 0$. Being more intuitive plausible, we choose to employ condition (38*β*).

- *Case* $\lambda A_H = 1$

The condition $\tau > 1$ implies that

$$A_L < \frac{Rq + h_{\min}}{(2-\lambda)h_{\min}} . \tag{37}$$

The right-hand side exceeds unity, as is required, given the imposition of

$$Rq > (1-\lambda)h_{\min} . \qquad (38) \ \|$$

We proceed to construct the mathematical conditions framing the equilibrium path along the interval $t \in [\tau, \infty)$. Upon noting that condition (26) is now met for both types of agents

$$V_t^{E,ND}(j) > V_t^{SR,D}(j) \qquad for\ j = L, H\ ,\ \forall t \in [\tau, \infty). \qquad (39)$$

Proposition 2 is rephrased to read as follows

Proposition 5 A private credit market for human capital investment is *sustainable* in the time interval $t \in [\tau, \infty)$ if and only if agents seeking credit are always offered a self-enforcing contract.

Proof

We have proved in the previous section that in this context with last-period consumption being exclusively determined by one's savings, the postulated feasibility of carrying out loan repayment, the use of a logarithmic utility function, and creditors backed by a system of strong legal rights, the contract arrangement (*q*, *R*) is self-enforcing (see *Proposition* 1). It follows that it takes only to impose the feasibility conditions for the two types to ensure that the educational credit market is privately sustainable. But feasibility is in fact established by *Proposition* 4 from period τ onward for both types. It follows that the proof of *Proposition* 5 entails the validity of[34]

$$A_L^{\delta}\, H_t^{1-\delta} > R\,q + h_{\min} \qquad \forall t \in [\tau, \infty), \qquad (32')$$

which is established as

$$A_L\, H_t \geq Rq + h_{\min} . \qquad \forall t \in [\tau, \infty). \qquad (34') \ \|$$

Corollary 1 The private credit market for educational investment is *sustainable* in each and all time periods of the interval $t \in [\tau, \infty)$ given the validity of condition (34').

The proof of existence of the equilibrium in which the entire population invests in education is enclosed in the following proposition

Proposition 6 A competitive equilibrium where the entire population acquires privately financed education exists on the condition of occurrence of the following requirements

- All types choose optimally to invest in individual improvement.

[34] It is only apparent that imposing the feasibility constraint for the low-type is sufficient to establish the analogous constraint for the high-ability agent.

- The credit market is privately sustainable.
- Individual saving be non-negative for both *H* and *L* types.

Proof

- We recall that the first condition constitutes the *participation constraint* for the borrower side. It is optimal to obtain education if and only if

$$V_t^{E,ND}(j) > V_t^{NE} \qquad \forall\, j = L, H\,, \tag{40}$$

which equivalently states

$$A_j^{\delta}\, H_t^{1-\delta} > Rq + h_{\min} \qquad \forall\, j = L, H\,. \tag{40'}$$

It is simply evident that imposing the individual rationality constraint for the low-type is sufficient to establish the analogous constraint for the high-ability agent. *Proposition* 4 establishes the validity of optimality condition (34') in the time interval $t \in [\tau, \infty)$, for the case of linear human capital technology.

- The second condition of the theorem involves the requirement that the credit market be sustainable. In light of *Corollary* 1 we recall that sustainability is established upon the validity of the optimality condition (34').
- In connection with the last condition, we impose the individual rationality constraints for the banking system, entailing that individual saving be positive for all types of agents. Invoking equation (8) we have

$$s_{2t+1}^{j*} = (1-\beta)\left(A_j^{\delta}\, H_t^{1-\delta} - Rq\right) > 0 \qquad \forall\, j \in \{L, H\}. \tag{41}$$

In analogy with our previous reasoning, we need only establish that saving be positive for the low-type agent. Evidently, this is already met in linear form under constraint (34').

It follows that the sole thing we must postulate to establish *Proposition* 6 for the case of the linear human capital technology is the optimality condition (34'). The truth of the remaining relationships is then logically inferred.‖

The dynamic evolution of the economy's aggregate stock of knowledge is governed by the first order non-linear difference equation:

$$H_{t+1} = \overline{A}\, H_t^{1-\delta} \qquad \forall \in [\tau, \infty), \tag{42}$$

where $\overline{A} \equiv (1-\lambda) A_L^{\delta} + \lambda A_H^{\delta}$. In the linear case $(i.e.\ \delta = 1)$ the solution to the difference equation is expressed by

$$H_t = \begin{cases} B^t H_\tau, & if\ B \neq 1 \\ H_\tau, & if\ B = 1 \end{cases} \qquad \forall t \geq \tau + 1 \tag{43}$$

where $B \equiv \lambda\, A_H + (1-\lambda)\, A_L$. The aggregate human capital H_τ is governed by equations (33).

IV. *Income distribution*

It has been the aim of this study to establish an analytic basis for the factual evidence lending truth to the Kuznets hypothesis. We show in the present section that this empirical phenomenon is proved within the theory, and is thus validated on the ground of acceptance of a mathematical proposition. Our proof procedure rests on the concept of *Lorenz ordering*, a notion which has been formally introduced by Chatterjee and Ravikumar (1999).

Following Kuznets (1955) we define the *income share* of a particular segment of society as the ratio of real per capita income within the specific group over the corresponding average for the entire population. In our context, individuals fall into two groups, with all agents within a class earning identical income. Therefore, it becomes relevant to specify the respective shares of the two distinct income groups (equivalently of the representative low- and high-type agents). The income share of group *j* at time period *t* is defined as $sh_t^j \equiv y_t^j / Y_t$, $\forall j \in \{L, H\}$. The economy-wide distribution is represented by the set of individual shares of all income classes. We define $\mathbf{sh}_{t+1}^{\zeta} = \{sh_{t+1}^{L\,\zeta}, sh_{t+1}^{H\,\zeta}\}$ for all $t \geq 0$, where ζ denotes the equilibrium type, $\zeta \in \{I, II, III\}$. Excerpted by Chatterjee and Ravikumar (1999) the definition of *Lorenz superiority* reads as follows

Definition 3 The low- and high-income groups (accordingly the entire measure of individuals) are arranged in a form of increasing order. Let there be two different economy-wide distributions of income shares, represented by $\mathbf{sh}$ and $\mathbf{sh}'$ respectively. It is said that $\mathbf{sh}$ is Lorenz superior to, or Lorenz dominates distribution $\mathbf{sh}'$ if $\sum_{j=L}^{\mu} \lambda_j\, sh_t'^{\,j} \leq \sum_{j=L}^{\mu} \lambda_j\, sh_t^j$ for all $\mu \in \{L, H\}$ and $t \in [0, \infty)$, with the inequality holding strictly for at least one μ.

The underlying logic of the notion of Lorenz superiority is that the distribution possessing this property exhibits lower inequality compared to any other income distribution. As is further brought out by Chatterjee and Ravikumar (1999), a Lorenz superior distribution is consistently ascribed a higher degree of equality by each and every conventionally used measure of inequality, such as the Gini coefficient, the coefficient of variation and the standard deviation of the logarithms.

We proceed to establish that the economy-wide income distribution in the state of poverty (equilibrium of type-*I*) Lorenz dominates the income distribution along the equilibrium path on which the economy develops due a segment of population engaging in human capital accumulation (type-*II* equilibrium). Subsequently, we demonstrate that the income distribution along the last stage of development, where the entire population participates in human capital accumulation (denoted equilibrium type-*III*), is Lorenz superior to the corresponding distribution of the precedent phase (equilibrium type-*II*).

Invoking the proposed definition by Kuznets (1955), and equations describing individual and average income, we obtain the following expression for the share of each income class in the poverty equilibrium

$$sh_{t+1}^{j\,I} = 1 \quad \forall j \in \{L, H\}, \forall t \in [0, r-1]. \tag{44}$$

In the phase of underdevelopment individuals of all types earn per capita income equal to the economy-wide average, $h_{\min}$. Genetic differences in learning aptitude *vanish* in the sense that they are not reflected in the income earning ability of agents. Absent heterogeneity in educational status, innate differences do not matter. The potential for differing earning productivities remains unrealized, with all workers being *trapped* in the choice of a single occupation, and therefore identical earnings. We call attention that perfect equality is an *endogenous* outcome in this context, caused by a *deficiency* in the economy's legislative system, namely the provision of insufficient legal power to financial institutions when faced with the possibility of default. Extreme credit rationing to educational investment, hence a missing credit market is an optimal response to a lack of commitment problem.

We have presupposed that adaptations in the legislative and judicial systems to accommodate *strong* legal protection of creditors are effective on period $t = r$, a date taken to be given exogenously.[35] The building of such *infrastructure* suffices to carry the economy out of its low income trap.[36] Along the growth path following such development, the different classes of agents earn respectively the income shares

$$sh_{t+1}^{L\,II} = h_{\min} \Big/ \left\{(1-\lambda)h_{\min} + \lambda A_H^{\delta} H_t^{II\,1-\delta}\right\} \qquad \forall t \in [r, \tau - 1]. \tag{45α}$$

$$sh_{t+1}^{H\,II} = A_H^{\delta} H_t^{II\,1-\delta} \Big/ \left\{(1-\lambda)h_{\min} + \lambda A_H^{\delta} H_t^{II\,1-\delta}\right\} \qquad \forall t \in [r, \tau - 1]. \tag{45β}$$

[35] We recall that the *switch* takes place in time period $t = r$, with this being the first period in which high-ability agents may invest in education. Since the return on education is realized on the subsequent date, $t = r + 1$ is the first relevant period for computing the income shares of the development stage *II*.

[36] Absent a system of public education, this is also the sole way to guide the economy out of stagnation.

Logically, the high-type agents represent the rich class earning the higher income share in the labor force. Applying the criterion of Lorenz superiority, we obtain that the income distribution $\mathbf{sh}_{t+1}^{I}$ Lorenz dominates the distribution $\mathbf{sh}_{t+1}^{II}$ under the condition that the following requirements be met

- $(1-\lambda)sh_{t+1}^{L\,II} \leq (1-\lambda)sh_{t+1}^{L\,I}$, (46)

implying

$$h_{\min} \leq A_H^{\delta}\, H_t^{II\,1-\delta} \quad \forall t \geq r\,, \tag{47}$$

and,

- $(1-\lambda)sh_{t+1}^{L\,II} + \lambda\, sh_{t+1}^{H\,II} \leq (1-\lambda)sh_{t+1}^{L\,I} + \lambda\, sh_{t+1}^{H\,I}$, (48)

whose elementary validation can be easily proved (each side equals unity). We are certain of the truth of relationship (47) given our assumption that the income of educated individuals may not to be exceeded by (or be equal to) the earnings of unskilled labor (*Assumption* 1).[37] We conclude that distribution $\mathbf{sh}_{t+1}^{I}$ is Lorenz superior to the income distribution $\mathbf{sh}_{t+1}^{II}$ on the condition of *Assumption* 1.

The logic underlying the conditions of Lorenz dominance relates to the growth pattern of the income shares of economic classes. We have established that the share of the group of low-type agents is smaller in each period of the interval $t \in [r, \tau - 1]$ compared to the group's share in dates of the stagnant equilibrium (relationship 46). Conversely, the income share of the high-type class is greater when the latter are able to invest in education compared to when they are constrained not to.[38] It immediately follows that condition (48) may be true only if the growth rate of the share of low-type class *upon transition* be larger (in absolute magnitude) to the respective rate of the share of talented ones. It is simple enough to show that this is in fact the case upon *Assumption* 1 being true. We have

$$g_T^{II}\left(sh_{t+1}^{L}\right) = \frac{\lambda\left(h_{\min} - A_H^{\delta}\, H_t^{II\,1-\delta}\right)}{(1-\lambda)h_{\min} + \lambda\, A_H^{\delta}\, H_t^{II\,1-\delta}} < 0\,, \tag{49}$$

and

[37] We note that under *Assumption* 1 relationship (46) holds as strict inequality only. Hence, all conditions are met for ascribing $\mathbf{sh}_{t+1}^{I}$ the property of Lorenz dominance on distribution $\mathbf{sh}_{t+1}^{II}$.

[38] This may be shown to be true upon inequality (47), hence on the already imposed *Assumption* 1.

$$g_T^{II}\left(sh_{t+1}^{H}\right)=\frac{(1-\lambda)\left(A_H^{\delta}\, H_t^{II\,1-\delta}-h_{\min}\right)}{(1-\lambda\,)h_{\min}+\lambda\, A_H^{\delta}\, H_t^{II\,1-\delta}}>0\,, \tag{50}$$

established to have a negative, and positive sign respectively on the basis of *Assumption* 1. In our chosen notation, $g_T^{\varsigma}\left(sh_{t+1}^{j}\right)$ denotes the rate of change of the income share of type-*j* agents as the economy attains equilibrium path $\varsigma\in\{II,III\}$ (*T* stands for transition). We write $g_T^{II}\left(sh_{t+1}^{j}\right)\equiv\left(sh_{t+1}^{j\,II}-sh_t^{j\,I}\right)/sh_t^{j\,I}$, where by definition $t\equiv r$. Low-ability individuals become *relatively* poorer in the transition to a higher stage of development, experiencing their income share to shrink. In addition, as the potential of high-aptitude agents has the opportunity to materialize, this class becomes richer in the economy's escape of poverty. We may easily prove that

$$(1-\lambda)g_T^{II}\left(sh_{t+1}^{L}\right)>\lambda\left|\,g_T^{II}\left(sh_{t+1}^{H}\right)\right|\ , \tag{51}$$

which holds upon *Assumption* 1 being true. The poor become poorer at a faster rate than the income of the rich is amplified. It is logically deduced that

$$\frac{y_{t+1}^{H\,II}}{y_{t+1}^{L\,II}}>\frac{y_t^{H\,I}}{y_t^{L\,I}}=1, \tag{52}$$

where $y_{t+1}^{j\,II}$, $j\in\{L,H\}$, is evaluated on the interval of date $t=r$, while $y_{t+1}^{j\,I}$ evaluated on $t\in[0,r-1]$ has a constant value. It is only evident that $sh_{t+1}^{H\,\zeta}/sh_{t+1}^{L\,\zeta}\equiv y_{t+1}^{H\,\zeta}/y_{t+1}^{L\,\zeta}$, $\forall\zeta\in\{I,II,III\}$, and $t\geq 0$. Clearly, it applies $y_{t+1}^{H\,II}/y_{t+1}^{L\,II}=A_H^{\delta}\,H_t^{II\,1-\delta}/h_{\min}$.

The pattern of worsening inequality prevails so long as the economy evolves along this *intermediate* stage of development. The poor become always poorer relatively to the society's average income, while the income of the rich is continuously amplified. It is plain that this is reflected in the direction of change of the respective income shares, as expressed by

$$g\left(sh_{t+1}^{L\,II}\right)=\frac{\lambda\,A_H^{\delta}\left(H_t^{II\,1-\delta}-H_{t+1}^{II\,1-\delta}\right)}{(1-\lambda\,)h_{\min}+\lambda\,A_H^{\delta}\,H_{t+1}^{II\,1-\delta}}<0 \qquad t\in[r+1,\tau-1]. \tag{53}$$

$$g\left(sh_{t+1}^{H\,II}\right)=\frac{(1-\lambda)h_{\min}\left(H_{t+1}^{II\,1-\delta}-H_t^{II\,1-\delta}\right)}{(1-\lambda\,)h_{\min}+\lambda\,A_H^{\delta}\,H_{t+1}^{II\,1-\delta}}>0 \qquad t\in[r+1,\tau-1]. \tag{54}$$

In fact, the same conclusion could be drawn from observing that the differential of per capita earnings widens in proportion to the rate of human capital accumulation realized in the time passed by. We point that

$$g\left(\frac{y_{t+1}^{H\,II}}{y_{t+1}^{L\,II}}\right)=\frac{H_t^{II\,1-\delta}-H_{t-1}^{II\,1-\delta}}{H_{t-1}^{II\,1-\delta}}>0\,,\qquad t\in[r+1,\tau-1], \tag{55}$$

where we define $g\left(\frac{y_{t+1}^{H\,\zeta}}{y_{t+1}^{L\,II}}\right)\equiv\frac{\left(y_{t+1}^{H\,\zeta}/y_{t+1}^{L\,\zeta}\right)-\left(y_t^{H\,\zeta}/y_t^{L\,\zeta}\right)}{y_t^{H\,\zeta}/y_t^{L\,\zeta}}$, along any equilibrium path ζ , $\zeta\in\{I,II,III\}$.

We have claimed in the previous section that due to perpetual growth educational investment may be consistent with optimal incentives eventually for the array of all types of agents. Income convergence is a characteristic that signals the economy has made its transition to this *advanced* stage in the growth path. Along this phase of development various economic classes earn the respective shares of aggregate output

$$sh_{t+1}^{j\,III}=A_j^{\delta}\big/(1-\lambda)A_L^{\delta}+\lambda\,A_H^{\delta}\quad\forall j\in\{L,H\},\ \forall t\in[\tau,\infty). \tag{56}$$

We establish that distribution $\mathbf{sh}_{t+1}^{II}$ is characterized by greater income inequality, in terms of Lorenz superiority, compared to the income distribution of the subsequent equilibrium phase, $\mathbf{sh}_{t+1}^{III}$. As it is known, the proof entails the requirement

- $(1-\lambda)sh_{t+1}^{L\,II}\le(1-\lambda)sh_{t+1}^{L\,III}$. (57)

Upon *Assumption* 1 relation (57) is satisfied as strict inequality.

$$h_{\min}<A_L^{\delta}\,H_t^{II\,1-\delta}. \tag{58}$$

It must further be met that

- $(1-\lambda)sh_{t+1}^{L\,II}+\lambda\,sh_{t+1}^{H\,II}\le(1-\lambda)sh_{t+1}^{L\,III}+\lambda\,sh_{t+1}^{H\,III}$, (59)

be always valid. It is intuitively clear that relation (59) is established upon the prerequisite that the decrease in the share of the rich class on the impact of transition be exceeded by the positive growth in the share of the poor. It is straightforward to show that

$$g_T^{III}\left(sh_{t+1}^{L}\right)=\frac{\lambda\,A_H^{\delta}\left(A_L^{\delta}\,H_t^{II\,1-\delta}-h_{\min}\right)}{h_{\min}\left[(1-\lambda)A_L^{\delta}+\lambda\,A_H^{\delta}\right]}>0\,, \tag{60}$$

while

$$g_T^{III}\left(sh_{t+1}^{H}\right)=\frac{(1-\lambda)\left(h_{\min}-A_L^{\delta}\,H_t^{II\,1-\delta}\right)}{H_t^{II\,1-\delta}\left[(1-\lambda)A_L^{\delta}+\lambda\,A_H^{\delta}\right]}<0\,, \tag{61}$$

are established to have a positive, and negative sign respectively on the basis of *Assumption* 1. We write $g_T^{III}\left(sh_{t+1}^{j}\right) \equiv \left(sh_{t+1}^{j\,III} - sh_t^{j\,II}\right) / sh_t^{j\,II}$, where by definition $t+1 \equiv \tau$. It is simple to show that

$$(1-\lambda) g_T^{III}\left(sh_{t+1}^{L}\right) > \lambda \left| g_T^{III}\left(sh_{t+1}^{H}\right) \right|, \tag{62}$$

with its validity resting upon *Assumption* 1.

It becomes evident from relations (60) and (61) that the income differential of the two classes shrinks as dynamics bring the economy on the more evolved stage in date $t = \tau + 1$. In consequence, we have

$$\frac{y_{t+1}^{H\,II}}{y_{t+1}^{L\,II}} > \frac{y_{t+1}^{H\,III}}{y_{t+1}^{L\,III}}, \tag{63}$$

with $y_{t+1}^{j\,II}$, $j \in \{L, H\}$, being evaluated on the interval $t \in [r, \tau - 1]$, while $y_{t+1}^{j\,III}$ on $t \in [\tau, \infty)$. We remark that the narrowing in earnings divergence consists of a discrete discontinuous jump occurring in consequence of the transition. The latter, we recall, is effected in the length of a sole time period, on date $\tau + 1$. Continuous, persistent fall in the inequality of income distribution does not follow ever sustained growth along this *final* stage of development. Economic classes claim each a constant share of the economy's output. Yet the rich remain always richer on this path, an event intrinsically plausible. It is only apparent that $y_{t+1}^{H\,III} / y_{t+1}^{L\,III} = A_H^{\delta} / A_L^{\delta}$, subject to no endogenous impact.

The evolution of inequality in aggregate distribution clearly exhibits a non-monotonic trend along the economy's path to development. The following theorem acknowledges the proposition formed by Kuznets (1955) as conclusively established.

Proposition 7 Should *Assumption* 1 be imposed, the relationship between inequality in the economy-wide income distribution and aggregate prosperity resembles an *inverse-U* shaped curve. Starting from perfect equality in a state of stagnation, income inequality exhibits a smooth upward trend as growth progressively takes off. Along this process, a critical threshold of development level is reached causing a qualitative change in dynamics. A sudden discontinuous fall in earnings' inequality is accompanied by a constant wage differential as unbounded growth is sustained perpetually.

We return our attention to the proposition of Chatterjee and Ravikumar (1999), that a Lorenz superior distribution is consistently ascribed a higher degree of equality by each and every conventionally used measure of inequality. The use of a simple example validates this argument, and lends confirmation to our conclusion that the dynamic pattern of the inequality of income distribution resembles an inverse-*U* curve. We choose to apply the widely used measure of *Gini* coefficient. The formal definition of the index reads as follows

$$G = \frac{1}{2N^2\overline{Y}} \sum_{j=1}^{J} \sum_{k=1}^{J} n_j n_k \left| y_j - y_k \right|. \qquad (64)$$

where we recall that N denotes the measure of aggregate population, $\overline{Y}$ represents the economy's average income, and J is the number of distinct incomes. Finally, subscripts j and k each represent an economic class, with $j, k \in \{1, 2, \ldots, J\}$. The population measure of each income group is hereby denoted by n_j.[39] In a recent study, Palivos and Yip (2007) have proved that for the simple case of $J = 2$, the aforementioned definition is written in the following simplified form

$$G = \frac{n_1}{N}\left(1 - \frac{y_1}{\overline{Y}}\right), \qquad (65)$$

where evidently $n_1 + n_2 = N$, and $y_1 = \min\{y_j\}_{j=1}^{J}$. In the context of our analysis, the *Gini* measure is clearly defined as

$$G_{t+1} = (1-\lambda)\left(1 - \frac{h_{t+1}^{L}}{H_{t+1}}\right). \qquad (65')$$

It can easily be proved that the value of the index in each equilibrium state is given by

$$G_{t+1}^{I} = 0 \qquad \forall t \in [0, r-1], \qquad (66)$$

$$G_{t+1}^{II} = \frac{\lambda(1-\lambda)\left(A_H^{\delta} H_t^{II\,1-\delta} - h_{\min}\right)}{(1-\lambda)h_{\min} + \lambda A_H^{\delta} H_t^{II\,1-\delta}} \qquad \forall t \in [r, \tau-1], \qquad (67)$$

with the latter being strictly positive on the account of *Assumption* 1. Further,

$$G_{t+1}^{III} = \frac{\lambda(1-\lambda)\left(A_H^{\delta} - A_L^{\delta}\right)}{(1-\lambda)A_L^{\delta} + \lambda A_H^{\delta}} \qquad \forall t \in [\tau, \infty], \qquad (68)$$

clearly, being a strictly positive constant. Theoretically, we confirm a positive growth measure on the account of *Assumption* 1, and positive growth for aggregate knowledge; conditions that do apply for both cases of linear and non-linear human capital technology. It has proved difficult within this framework to obtain a definable prediction of the rate of change of the growth measure of the *Gini* coefficient in the course of the *intermediate* stage of development. We have

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[39] The definition is excerpted from Debraj Ray (1998).

$$g\left(G_{t+1}^{II}\right)=\frac{A_H^{\delta}\,h_{\min}\left(H_t^{1-\delta}-H_{t-1}^{1-\delta}\right)}{\left(A_H^{\delta}\,H_{t-1}^{1-\delta}-h_{\min}\right)H_t} \qquad \forall t\in\left[r+1,\tau-1\right]. \tag{69}$$

where we have defined $g\left(G_{t+1}^{II}\right)\equiv\left(G_{t+1}^{II}-G_t^{II}\right)/G_t^{II}$, and $\delta\in(0,1)$. In the linear case, the result is expressed as following

$$g\left(G_{t+1}^{II}\right)=\frac{A_H\,h_{\min}\;H_{t-1}^{II}}{\left(A_H\,H_{t-1}-h_{\min}\right)H_t^{II}}\,g\left(H_t^{II}\right) \qquad \forall t\in\left[r+1,\tau-1\right]. \tag{70}$$

where in analogous manner we have defined $g\left(H_t^{II}\right)\equiv\left(H_t^{II}-H_{t-1}^{II}\right)/H_{t-1}^{II}$. Using the solution for H_t as given by equation (31), the growth result is written

$$g\left(G_{t+1}^{II}\right)=\frac{\left(A_H-1\right)\left(\lambda A_H-1\right)^2\left(\lambda A_H\right)^t}{\Omega_1+\Omega_2\left(\lambda A_H\right)^t+\Omega_3\left(\lambda A_H\right)^{2t}} \qquad \forall t\in\left[r+1,\tau-1\right], \tag{70'}$$

where $\Omega_1\equiv A_H\left(1-\lambda\right)^2+\lambda\left(1-A_H\right)^2A_H^{-1}-\left(1-\lambda A_H\right)\left(1-\lambda\right)$,
$\Omega_2\equiv A_H\,\lambda\left(1-\lambda\right)\left(1-A_H\right)+\lambda\left(1-\lambda A_H\right)\left(A_H-1\right)$, and $\Omega_3\equiv\lambda^2\left(1-A_H\right)^2$. Performing a simulation analysis would enable us to sign the expression, and allow us to conclude about the form of upward trend of the inequality of income distribution.

It is evident that the following holds true

$$G_{t+1}^{I}<G_{t+1}^{II}, \tag{71}$$

and

$$G_{t+1}^{III}<G_{t+1}^{II}, \tag{72}$$

with the latter being valid on the basis of *Assumption* 1. The results lead us to infer once again an inverted-*U* pattern of evolution in personal income inequality. Due to strong legal rights be established for credit institutions, growth prospects start to materialize carrying the economy to a path of higher and worsening inequality. Society escapes a poverty loophole, albeit a state of perfect equality. It can be logically guaranteed that inequality will take eventually a downward jump to a fixed computable level at a high measure of probability (see equation 68). We have remarked upon the existence of the critical time period in which this occurs in *Proposition* 4.

The *Kuznets Curve* that our theory implies is presented in the following diagram:

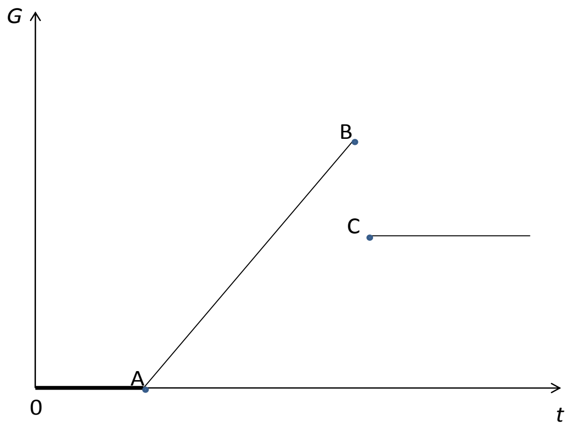

Figure I.1: *Kuznets Curve (benchmark version of the model)*

It is evident to the reader that points *A* and *B* correspond to the critical time periods *r* and τ, respectively, at which the transition takes place to a new equilibrium path. Point *C* marks the start of the *advanced* phase of development at period $\tau+1$.

V. *Extension to general equilibrium*

In our previous environment financial intermediaries had access to a hypothesized credit market *outside* of the economy. The sole participants in this market were financial institutions, able to borrow and invest any amount at an exogenously fixed interest rate. It is within our scope to extend the preceding analysis in a way that allows the interest rate being endogenously determined. Equilibrating forces in the credit market require that loan demand be equated to credit supply. Making use of Walras's law, we choose to employ the economy's equilibrium resource constraint, that aggregate investment be equal to domestic saving.

We proceed with a concise presentation of our theory, as is reformulated to account for the endogeneity of the interest rate, *R*. The model's previous analytical structure is followed in near precision, while the linguistic accompaniment of mathematics is justly omitted.

The budget constraint in the working period of life is now given by the expression

$$c_{2t+1}^{j}=\begin{cases} A_j^{\delta}\,H_t^{1-\delta}-R_{t+1}\,q-s_{2t+1}^{j} & if\;\; q>0 \\ h_{\min}-s_{2t+1}^{j} & if\;\; q=0 \end{cases}. \tag{73}$$

Accordingly, consumption in the retirement age is given by

$$c_{3t+2}^{j} = R_{t+2}\, s_{2t+1}^{j}\,. \tag{74}$$

The optimization problem yields that individual optimal saving is

$$s_{2t+1}^{j*} = \begin{cases} (1-\beta)\left(A_j^{\delta}\, H_t^{1-\delta} - R_{t+1}\, q\right) & if\;\; q > 0 \\ (1-\beta)\, h_{\min} & if\;\; q = 0 \end{cases} \quad \forall j = L, H\,. \tag{75}$$

Substituting for optimal saving in equations (73) and (74) we obtain the respective expressions for optimal second- and third period consumption

$$c_{2t+1}^{j*} = \begin{cases} \beta\left(A_j^{\delta}\, H_t^{1-\delta} - R_{t+1}\, q\right) & if\;\; q > 0 \\ \beta\, h_{\min} & if\;\; q = 0 \end{cases} \quad \forall j \in \{L, H\}. \tag{73'}$$

$$c_{3t+2}^{j*} = \begin{cases} (1-\beta) R_{t+2}\left(A_j^{\delta}\, H_t^{1-\delta} - R_{t+1}\, q\right) & if\;\; q > 0 \\ (1-\beta) R_{t+2}\, h_{\min} & if\;\; q = 0 \end{cases} \quad \forall\, j \in \{L, H\}. \tag{74'}$$

State of poverty The preceding analysis established that a state of underdevelopment, accompanied by perfect equality in low income earnings, is a prospect actualized indefinitely when credit institutions are entitled to limited rights in claiming debt repayment. Financial entities lack the incentive to provide credit for human capital investment, with the consequence of agents' preclusion from the opportunity to privately financed education. In that environment, the possibility of saving for retirement age is served through the private financial system, construed to be composed of a form of deposit institutions. Saving yields a positive rate of return, modeled as an exogenous fixed variable, which may not be otherwise endogenously determined.[40]

A segment of the society invests We proceed to generalize within the context of general equilibrium the path along which potential ever sustained growth is realized. Owing to appropriate transformations in the economy's legislative system, a prior missing market for human capital investment may function on time period r, where $r \geq 2$.

The postulated heterogeneity on innate aptitude level becomes critical for individual decision making, and consequently for the economy's growth pattern and inequality of income distribution. We redefine formally

[40] An alternative way to retain the possibility of saving for retirement age would be to postulate the existence of a storage technology, taken to provide a positive return on investment. The conception of such technology ought to be considered an arbitrarily chosen device, with the meaning to supplement the role of absent saving institutions.

individual heterogeneity in income earning ability, recognizing that the interest rate is now endogenously determined in each period. *Definition* 1 is mathematically expressed as follows:

Assumption 3 An agent born at time period *t* is said to be of *high type* if and only if

$$A_H^{\delta}\, H_t^{1-\delta} > R_{t+1}^{*II}\, q \qquad \forall t \geq 0. \tag{α}$$

On the other hand, it is not feasible for *low type* agents to carry out their contract commitment, however honest in intention. Mathematically, this reads into

$$A_L^{\delta}\, H_t^{1-\delta} < R_{t+1}^{*II}\, q \qquad \forall t \geq 0. \tag{β}$$

The variable R_{t+1}^{*II} expresses the endogenous value of the interest rate as determined by the economy's equilibrium resource constraint. The latter reads that aggregate saving be equal to the private demand for investment, which solely consists of the credit financing human capital accumulation. Formally expressed, we have

$$S_{2\,t+1}^{II} = \lambda\, q \qquad \forall t \geq r, \tag{76}$$

where S_{2t+1}^{II} denotes domestic private saving accumulated in the working period of life from members of generation *t*. Evidently, $\lambda\, q$ represents the private demand for educational investment. On the other side, aggregate saving is to be defined as

$$S_{2\,t+1}^{II} = (1-\lambda)(1-\beta)h_{\min} + \lambda(1-\beta)\left(A_H^{\delta}\, H_t^{II\,1-\delta} - R_{t+1}^{*II}\, q\right) \qquad \forall t \geq r. \tag{77}$$

We obtain that the equilibrium expression of the interest rate price is given by

$$R_{t+1}^{*II} = \frac{(1-\lambda)h_{\min}}{\lambda\, q} + \frac{A_H^{\delta}\, H_t^{II\,1-\delta}}{q} - \frac{1}{1-\beta} \qquad \forall t \geq r. \tag{78}$$

The function conveys that the rate of return on financial assets is derived by the technology of human capital accumulation.

We confine our attention to establishing the proof of *Proposition* 3, which forms the essence of this analysis. Similarly worded, the statement of the theorem is the following

Proposition 8 A competitive equilibrium with a subset of population acquiring privately financed education exists on the condition of occurrence of the following requirements

- A positive measure of individuals optimally choose to obtain education

- The credit market is sustainable
- Savings be non-negative for both *H* and *L* types.

Proof

- The measure of high-type agents must be induced to *participate* in the contract arrangement. This translates to mean that the contract must offer a high-ability individual at least the utility level she would obtain if she chose to remain unskilled. In other words,

$$V_t^{E,ND}(H) > V_t^{NE,II} \qquad \forall t \geq r\,, \tag{79}$$

where it holds that

$$V_t^{E,ND}(j) = \ln\left\{\beta^{\beta}(1-\beta)^{1-\beta} R_{t+2}^{*\,1-\beta}\left(A_j^{\delta} H_t^{1-\delta} - R_{t+1}^{*}\, q\right)\right\} \quad \forall j \in \{L,H\}. \tag{80}$$

$$V_t^{NE}(j) \equiv V_t^{NE} = \ln\left\{\beta^{\beta}(1-\beta)^{1-\beta} R_{t+2}^{*\,1-\beta}\, h_{\min}\right\} \quad \forall j \in \{L,H\}. \tag{81}$$

Relation (79) is expressed as

$$A_H^{\delta} H_t^{II\,1-\delta} > h_{\min} + q\, R_{t+1}^{*} \qquad \forall t \geq r\,, \tag{79'}$$

which, evaluated in period $t = r$ yields

$$A_H^{\delta} h_{\min}^{1-\delta} > h_{\min} + q\, R_{t+1}^{*\,II} \qquad \forall t \geq r\,. \tag{79''}$$

Evidently, it suffices to impose relationship (79'') to ensure the validity of (79') in all periods $t > r$.

- Drawing upon *Proposition* 2 we transfer the conclusion that the postulated definitions of agent types (*Assumption* 3) count as proof of the sentence that the educational credit market is privately sustainable.
- The individual rationality constraint for the members of the financial system amounts to imposing that individual saving is positive for both types. Invoking the optimal saving function (equation 8), we have that

$$s_{2t+1}^{H\,*} = (1-\beta)\left(A_H^{\delta} H_t^{1-\delta} - R_{t+1}^{*\,II}\, q\right) > 0\,, \tag{82}$$

which is strictly positive on the basis of the definition of the high-type agent (*Assumption* 3α). With respect to the low-type individuals saving is represented by

$$s_{2t+1}^{L\,*} = (1-\beta)\, h_{\min} > 0 \qquad \forall t \geq 0\,, \tag{83}$$

which is strictly positive for $\beta \in (0,1)$, and $h_{\min} > 0$.

The proof of *Proposition* 8 clearly consists of imposing the definitional condition *Assumption* 3β, as well as the constraint (79''), along with the equilibrium expression of the interest rate (equation 77). The truth of the remaining relations is inferred by simple reasoning.‖

All types acquire education In analogy with the analysis in the core version the following theorem proves the transition of the economy to a higher stage of development, where education is accommodated for all types of agents. The following is a restatement of *Proposition* 4, carried in the general equilibrium context

Proposition 9 There exists a time period $\tilde{\tau} > 0$, where $\tilde{\tau} \in [r+1, \infty)$, in which the income realization of educated low-type agents exceeds the threshold level that defines education the optimal choice. This reads as follows

$$A_L^{\delta} H_t^{1-\delta} \begin{cases} \geq R_{t+1}^{*} q + h_{\min} & \forall t \in [\tilde{\tau}, \infty) \\ < R_{t+1}^{*} q + h_{\min} & \forall t \in [0, \tilde{\tau} - 1] \end{cases}. \tag{84}$$

If we limit ourselves to the linear case $(\delta = 1)$ we can prove that $\tilde{\tau}$ is defined as

$$\tilde{\tau} = \begin{cases} \dfrac{\ln[\tilde{\Theta}]}{\ln[\lambda A_H]}, & if\ \lambda A_H > 1 \\ \\ \dfrac{R_{t+1}^{*III} q + h_{\min}(1 - A_L)}{A_L h_{\min}(1-\lambda)}, & if\ \lambda A_H = 1 \end{cases}, \tag{85}$$

with $\tilde{\Theta} \equiv \dfrac{\left(R_{t+1}^{*III} q + h_{\min}\right)(1 - \lambda A_H) - A_L h_{\min}(1-\lambda)}{A_L h_{\min} \lambda (1 - A_H)}$. The expression is identical to the definition of τ in the baseline version with the obvious exception that *R* is now endogenously determined.

Proof

The reasoning of the proof is completely analogous to that of *Proposition* 4. Relationship (75) yields in linear form

$$A_L H_t \geq R_{t+1}^{*III} q + h_{\min} \qquad \forall t \in [\tilde{\tau}, \infty). \tag{86}$$

We use the solution of aggregate human capital stock (equations 31) to substitute for H_t. Deriving expression (85) is then a straightforward task.

We can prove the existence of time period $\tilde{\tau}$ only on the condition that the latter is greater to unity. More precisely, the theorem is true if

- *Case* $\lambda A_H > 1$

$$\tilde{\Theta} > \lambda A_H \,, \tag{87}$$

which leads to a standard second-order polynomial

$$\left(\lambda^2 A_L h_{\min}\right) A_H^2 - \lambda \left[R_{t+1}^{*III} q + h_{\min}\left(1 + \lambda A_L\right)\right] A_H + R_{t+1}^{*III} q + h_{\min}\left[1 - A_L\left(1 - \lambda\right)\right] > 0 \,. \tag{87'}$$

The expression is positive insofar as, either

$$A_H < \frac{R_{t+1}^{*III} q + h_{\min}\left(1 + \lambda A_L\right) - \sqrt{\Omega}}{2\lambda A_L h_{\min}} \equiv A_H^1 \,, \tag{88α}$$

or, alternatively

$$A_H > \frac{R_{t+1}^{*III} q + h_{\min}\left(1 + \lambda A_L\right) + \sqrt{\Omega}}{2\lambda A_L h_{\min}} \equiv A_H^2 \,, \tag{88β}$$

where we note that $\Omega \equiv \left[R_{t+1}^{*III} q + h_{\min}\left(1 + \lambda A_L\right)\right]^2 - 4 A_L h_{\min}\left[R_{t+1}^{*III} q + h_{\min}\left(1 - A_L\left(1 - \lambda\right)\right)\right] \geq 0$, and $A_H^1 > 0$.

Being more intuitive plausible, we choose to employ condition (88*β*).

- *Case* $\lambda A_H = 1$

The condition $\tilde{\tau} > 1$ implies that

$$A_L < \frac{R_{t+1}^{*III} q + h_{\min}}{\left(2 - \lambda\right) h_{\min}} \,. \tag{89}$$

The right-hand side exceeds unity, as is required, given the imposition of

$$R_{t+1}^{*III} q > \left(1 - \lambda\right) h_{\min} \,. \tag{90}$$ ‖

We recall, R_{t+1}^{*} expresses the endogenous value of the interest rate as being determined by the economy's equilibrium resource constraint. Along the time interval $t \in \left[\tilde{\tau}, \infty\right)$, the latter is defined to read as follows

$$S_{2\,t+1}^{III} = q \qquad \forall t \in \left[\tilde{\tau}, \infty\right). \tag{91}$$

where we recall that S_{2t+1}^{III} denotes the domestic private saving accumulated in the working period of life from members of generation *t*. The latter is defined as

$$S_{2\,t+1}^{III} = (1-\beta)\{(1-\lambda)A_L^{\delta} + \lambda A_H^{\delta}\}H_t^{1-\delta} - (1-\beta)q R_{t+1}^{*} \qquad \forall t \in [\tilde{\tau}, \infty). \tag{92}$$

The private demand for educational investment is expressed by variable q. It is straightforward to obtain the equilibrium expression for the interest rate, being defined as

$$R_{t+1}^{*\,III} = \frac{\{(1-\lambda)A_L^{\delta} + \lambda A_H^{\delta}\}H_t^{1-\delta}}{q} - \frac{1}{1-\beta} \qquad \forall t \in [\tilde{\tau}, \infty). \tag{93}$$

The proof of existence of the equilibrium state along which growth is supported by human capital investment of all types is enclosed in a restatement of *Proposition* 6

Proposition 10 A competitive equilibrium where the entire population acquires privately financed education exists on the condition of occurrence of the following requirements

- All types optimally choose to invest in individual improvement
- The credit market is privately sustainable
- Individual saving be non-negative for both *H* and *L* types

Proof

- Individuals of either type must be induced to engage in human capital investment. This is guaranteed only insofar as

$$V_t^{E,ND}(j) > V_t^{NE} \qquad \forall\, j = L, H\,, \forall t \in [\tilde{\tau}, \infty), \tag{94}$$

which is equivalent to

$$A_j^{\delta}\, H_t^{1-\delta} > R_{t+1}^{*\,III}\, q + h_{\min} \qquad \forall\, j = L, H\,, \forall t \in [\tilde{\tau}, \infty), \tag{94'}$$

given that $V_t^{E,ND}(j)$, and V_t^{NE} are defined by equations (80) and (81) respectively. Evidently, it suffices to impose the sole relation

$$A_L^{\delta}\, H_t^{1-\delta} > R_{t+1}^{*\,III}\, q + h_{\min} \qquad t = \tilde{\tau}\,. \tag{94''}$$

- In light of *Corollary* 1, the sustainability of credit market is established upon the validity of condition (94'').
- In connection with the last condition, we impose once more the individual rationality constraints for the banking system (equations 11). Invoking equation (8), we obtain

$$s_{2t+1}^{j*} = (1-\beta)\left(A_j^{\delta}\, H_t^{1-\delta} - R_{t+1}^{*\,III}\, q\right) > 0 \qquad \forall\, j \in \{L, H\}, \forall t \in [\tilde{\tau}, \infty) \tag{95}$$

We need only establish that saving be positive for the low-type agent, which evidently is met under condition (94").

It follows that the sole thing we must postulate to establish *Proposition* 10 is inequality (94"). The truth of the remaining relations is logically inferred.║

VI. *Higher degree of heterogeneity*

An appropriate extension of the basic construct of the model would be to augment the set of values in the domain of innate ability, A_j. Such a generalization is, in and of itself, a noteworthy task in that we lay down the theory in higher mathematical abstraction. Yet its practical value is that it provides us with a way of obtaining a *smooth* inverted-*U* curve. Taking heterogeneity to the highest degree of generality, we postulate that the domain of variable A_j forms a district measure of (countable) infinite types. In specific, we assert that the measure of heterogeneous types is defined on the bounded interval $\forall j \in [1,\dots,J]$, where $J \geq 2$.[41] Each class of type-*j* individuals constitutes a fraction λ_j of the population measure, $N=1$. It is evident that $\sum_{j=1}^{J}\lambda_j = 1$.

It may easily be seen that the structure of the model set out in section *II*, as well as the analysis on the stationary equilibrium path, becomes no different when the general case $J > 2$ is applied.[42] Carrying not an unfruitful repetition, we proceed with the analysis on equilibrium growth.

We recall that our definition of *individual type* is intrinsically connected to an agent's innate aptitude towards knowledge acquisition, a factor being genetically, or otherwise exogenously determined. Nevertheless, human capital productivity does not critically determine the ability to carry out one's contract commitment.[43] Being of a certain type is accompanied by no idiosyncratic feature determining the feasibility of loan repayment. The definition of the latter concept formally reads as follows[44]

Definition 4 Loan repayment is *feasible* for an agent born in time *t* if the following holds to be true

$$A_j^{\delta} H_t^{1-\delta} > Rq\,. \tag{96}$$

The following assumption is adopted

[41] We postulate that for any two cardinal numbers l and m of the set $j \in [1,2,\dots,J]$, with $1 \leq l < m \leq J$, the corresponding members in the set of heterogeneous abilities $A_j \in [A_1, A_2, \dots, A_m]$ are of the same cardinal ordering, i.e. $A_l < A_m$.

[42] The present analysis refers to the case of exogenously determined interest rate. Hence, it is relevant to observe equations (1) to (23).

[43] This was a natural feature of the simple two-type case. In that setting, differentiation in income earning ability was inescapably synonymous to critical differences in the feasibility of carrying out contract obligations.

[44] The definition of the concept has already been introduced in *Assumption* 2.

Assumption 4 Individuals of type $j \in [1,\ldots,k]$, where $k \in [1,\ldots,J-1]$, are discerned by an income earning ability as low as not in the least covering debt repayment. On the other hand, the earned income of an agent of type $j \in [k+1,\ldots,J]$ supports her honoring of debt liability. Hence, we explicitly postulate

$$A_j^{\delta} H_t^{1-\delta} < Rq \qquad j \in [1,\ldots,k]. \tag{α}$$

$$A_j^{\delta} H_t^{1-\delta} > Rq \qquad j \in [k+1,\ldots,J]. \tag{β}$$

Assumption 4(α) says that investment in human capital does not pay off for types $j \in [1,\ldots,k]$ if one is to remain committed to contract liability. Default on debt is an inescapable consequence, irrespective of an inherent honest intention. The assumption is reducible to the following expression

Assumption 4'

$$A_k^{\delta} H_t^{1-\delta} < Rq \tag{α}$$

$$A_{k+1}^{\delta} H_t^{1-\delta} > Rq \tag{β}$$

We are now able to prove, by means of what has already been said, that an equilibrium state may exist with the economy being placed on a path of ever-sustained growth. As has been stated previously, this potential is realized as a result of the private financing for education being made feasible, if only initially for the segment of society with relatively high investment return (namely, the types $j \in [k+1,\ldots,J]$). Similarly worded, the foregoing theorem forms an exact generalization of *Proposition* 3 to the multiple *J*-type case.

Proposition 11 A competitive equilibrium with a subset of the population acquiring privately financed education exists on the condition of occurrence of the following requirements

- A positive measure of individuals optimally choose to obtain education,
- The credit market is sustainable,
- Savings be non-negative for all agent types.

Proof

- The *participation constraint* for the borrower side consists of the following condition

$$V_t^{E,ND}(j) > V_t^{NE} \qquad \forall t \geq r\,,\ j \in [k+1,\ldots,J], \tag{97}$$

where we recall, $V^{E,ND}(j)$, and V^{NE} are given by equations (18) and (19) respectively. Relation (88) is satisfied so long as

$$A_j^{\delta} H_t^{II\,1-\delta} > h_{\min} + Rq \qquad \forall t \geq r\,,\ j \in [k+1,\ldots,J], \tag{97'}$$

which, evaluated in period $t = r$, for type $j = k+1$ yields

$$A_{k+1}^{\delta}\, h_{\min}^{1-\delta} > h_{\min} + Rq. \tag{97''}$$

Evidently, it suffices to impose relationship (97") to ensure the validity of (97') for all types $j \in [k+1,\dots,J]$, in each and all time periods $t \geq r$.

- Drawing on *Proposition* 2, it is purely logically proved that the financial market for human capital investment is privately sustainable. The requirement of proof is fully satisfied upon the postulated definitions of *Assumption* 4'.
- In order to prove that it is individually rational for credit entities to engage in loan provision we need to impose that saving be positive for all types of agents. Invoking the optimal saving function (equation 8) we obtain

$$s_{2t+1}^{j\,*} = (1-\beta)\left(A_j^{\delta}\, H_t^{II\,1-\delta} - Rq\right) > 0 \qquad \forall t \geq r,\ j \in \{k+1,\dots,J\}, \tag{98}$$

which is strictly positive on the basis of *Assumption* 4'(β). With respect to the individuals of relatively low productivity saving is given by

$$s_{2t+1}^{j\,*} = (1-\beta)\, h_{\min} > 0 \qquad \forall t \geq r,\ j \in \{1,\dots,k\}, \tag{99}$$

obviously being strictly positive for $\beta \in (0,1)$, and $h_{\min} > 0$. In summary, the proof of *Proposition* 11 requires the validity of *Assumption* 4'(α), and of constraint (97"). The truth of the remaining relations is then logically inferred.‖

The dynamic evolution of the society's stock of human capital along this equilibrium path is governed by the first order non-linear difference equation

$$H_{t+1} = h_{\min} \sum_{j=1}^{k} \lambda_j + H_t^{II\,1-\delta} \sum_{j=k+1}^{J} \lambda_j A_j^{\delta} \qquad \forall t \geq r. \tag{100}$$

The solution to the linear form $(i.e.\ \delta = 1)$ is described as

$$H_t = \begin{cases} h_{\min}\left[(1-\varphi)\theta^t + \varphi\right] & \text{if } \theta \equiv \sum_{j=k+1}^{J} \lambda_j A_j \neq 1 \\ h_{\min}\left(1 + t\sum_{j=1}^{k} \lambda_j\right) & \text{if } \theta \equiv \sum_{j=k+1}^{J} \lambda_j A_j = 1 \end{cases} \qquad \forall t \geq r+1. \tag{101}$$

where we define $\varphi \equiv \frac{\sum_{j=1}^{k}\lambda_j}{1-\theta}$.

We prove the existence of a critical level of development which, once reached, type *k* has an optimal incentive to invest in education.

Proposition 12 There exists a time period $\tau_1 > 0$, where $\tau_1 \in [r+1, \infty)$, in which the income realization of educated type-*k* agents exceeds the threshold level that defines education the optimal choice. In other words,

$$A_k^{\delta} H_t^{1-\delta} \begin{cases} \geq Rq + h_{\min} & \forall t \in [\tau_1, \infty) \\ < Rq + h_{\min} & \forall t \in [0, \tau_1 - 1] \end{cases}. \tag{102}$$

Considering the linear human capital technology, τ_1 is defined as

$$\tau_1 = \begin{cases} \frac{\ln[\Phi]}{\ln[\theta]}, & if\ \theta > 1 \\ \frac{Rq + h_{\min}(1 - A_k)}{A_k h_{\min} \sum_{j=1}^{k} \lambda_j}, & if\ \theta = 1 \end{cases}, \tag{103}$$

with $\Phi \equiv \frac{Rq + h_{\min}(1 - \varphi A_k)}{A_k h_{\min}(1 - \varphi)}$.

Proof

Relationship (93) is written in linear form

$$A_k H_t \geq Rq + h_{\min}. \tag{104}$$

We use the solution of aggregate human capital stock as given by (101), to substitute for H_t. Equations (103) are then derived in a straightforward manner.

The requirement must be imposed that τ_1 be greater than one. More precisely, it must hold that

- $$\frac{Rq + h_{\min}(1 - \varphi A_k)}{A_k h_{\min}(1 - \varphi)} > \sum_{j=k+1}^{J} \lambda_j A_j \qquad if\ \theta > 1. \tag{105}$$

- $A_k < \dfrac{Rq + h_{\min}}{\left(1 + \sum_{j=1}^{k} \lambda_j\right) h_{\min}}$. (106) ‖

We prove the following theorem

Proposition 13 A competitive equilibrium exists in each and all time periods $t \in [\tau_1, \infty)$ having the following characteristics: the measure of population with learning abilities ranging in the interval $A_j \in [A_k, A_J]$ acquire privately financed education, whereas the remaining subset with abilities $A_j \in [A_1, A_{k-1}]$ choose to remain unskilled. The existence of the equilibrium is established on the condition of occurrence of the following requirements

- The measure of the population with ability types $j \in [k, J]$ optimally choose to obtain education.
- The credit market is sustainable.
- Savings be non-negative for all agent types.

Proof

- As has already been mentioned, the *participation constraint* for the borrower side consists of the condition

$$V_t^{E,ND}(j) > V_t^{NE} \qquad \forall t \geq \tau_1,\, j \in [k, \ldots, J], \tag{107}$$

Drawing upon equations (18) and (19), we obtain that relation (107) is satisfied so long as

$$A_j^{\delta} H_t^{1-\delta} > h_{\min} + Rq \qquad \forall t \geq \tau_1,\, j \in [k, \ldots, J], \tag{107'}$$

Evidently, it suffices to impose relationship (107') for type $j = k$ to ensure its validity for all remaining types $j = k+1, \ldots, J$. Evaluated in the linear case, expression (107') yields

$$A_k > \frac{h_{\min} + Rq}{H_t} \qquad \forall t \geq \tau_1. \tag{107''}$$

- Drawing on *Proposition* 2, it is logically proved that the financial market for human capital investment is privately sustainable. The requirement of proof is fully satisfied upon the feasibility conditions

$$A_{k-1}^{\delta} H_t^{1-\delta} < Rq \qquad \forall t \geq \tau_1, \tag{108}$$

$$A_k^{\delta} H_t^{1-\delta} > Rq \qquad \forall t \geq \tau_1. \tag{109}$$

- Finally, we need to impose that saving is positive for all types of agents. Invoking the optimal saving function (equation 8) we have

$$s_{2t+1}^{j*} = (1-\beta)\left(A_j^{\delta} H_t^{II\,1-\delta} - Rq\right) > 0 \qquad \forall t \geq r\,,\; j \in \{k\ldots, J\}, \tag{110}$$

which is strictly positive on the basis of condition (107"). With respect to the individuals of relatively low productivity, saving is given by

$$s_{2t+1}^{j*} = (1-\beta)\, h_{\min} > 0 \qquad \forall t \geq r\,,\; j \in \{1,\ldots, k-1\}, \tag{111}$$

obviously being strictly positive for $\beta \in (0,1)$, and $h_{\min} > 0$. In summary, the proof of *Proposition* 13 requires the validity of conditions (107"), and (108). The truth of the remaining relations is then logically inferred.$\|$

The dynamic evolution of the society's stock of human capital along the time path $t \in \geq \tau_1$ is governed by

$$H_{t+1} = h_{\min} \sum_{j=1}^{k-1} \lambda_j + H_t^{1-\delta} \sum_{j=k}^{J} \lambda_j A_j^{\delta} \qquad t \in \geq \tau_1\,. \tag{112}$$

The solution to the linear form of difference equation (112) $(i.e.\ \delta = 1)$ is described by

$$H_t = \begin{cases} h_{\min}\left[(1-\tilde{\varphi})\tilde{\theta}^{\,t} + \tilde{\varphi}\right] & \text{if } \tilde{\theta} \equiv \sum_{j=k}^{J} \lambda_j A_j > 1 \\ & \qquad\qquad \forall t \geq \tau_1 + 1\,. \\ h_{\min}\left(1 + t\sum_{j=1}^{k-1} \lambda_j\right) & \text{if } \tilde{\theta} \equiv \sum_{j=k}^{J} \lambda_j A_j = 1 \end{cases} \tag{113}$$

where we define $\tilde{\varphi} \equiv \dfrac{\sum_{j=1}^{k-1} \lambda_j}{1-\tilde{\theta}}$.

The following theorem proves that per capita income reaches a critical threshold which, once surpassed, agents of type *k*-1 optimally choose to invest in education.

Proposition 14 Let there exist a time period $\tau_2 > 0$, with $\tau_2 \in [\tau_1 + 1, \infty)$. The income realization of educated individuals of type *k*-1 exceeds the threshold level that defines education the optimal choice. In other words,

$$A_{k-1}^{\delta} H_t^{1-\delta} \begin{cases} \geq Rq + h_{\min} & \forall t \in [\tau_2, \infty) \\ < Rq + h_{\min} & \forall t \in [0, \tau_2 - 1] \end{cases}. \tag{114}$$

Considering the linear human capital technology, τ_2 is defined as

$$\tau_2 = \begin{cases} \dfrac{\ln\left[\tilde{\Phi}\right]}{\ln\left[\tilde{\theta}\right]}, & if\ \tilde{\theta} > 1 \\ \dfrac{Rq + h_{\min}\left(1 - A_{k-1}\right)}{A_{k-1} h_{\min} \sum_{j=1}^{k-1} \lambda_j}, & if\ \tilde{\theta} = 1 \end{cases}, \tag{115}$$

with $\tilde{\Phi} \equiv \dfrac{Rq + h_{\min}\left(1 - \tilde{\varphi} A_{k-1}\right)}{A_{k-1} h_{\min}\left(1 - \tilde{\varphi}\right)}$.

Proof

Equation (115) is derived by following the same procedure as in *Proposition* 12. We write relationship (114) in linear form

$$A_k H_t \geq Rq + h_{\min}. \tag{116}$$

Using the solution of aggregate human capital stock as given by (110) to substitute for H_t, we obtain the expression for τ_2 (115). Once again, the requirement must be imposed that τ_2 be greater to unity. Precisely, it must be

- $$\frac{Rq + h_{\min}\left(1 - \tilde{\varphi} A_{k-1}\right)}{A_{k-1} h_{\min}\left(1 - \tilde{\varphi}\right)} > \sum_{j=k}^{J} \lambda_j A_j \qquad if\ \tilde{\theta} > 1, \tag{117}$$
- $$A_{k-1} < \frac{Rq + h_{\min}}{\left(1 + \sum_{j=1}^{k-1} \lambda_j\right) h_{\min}}. \tag{118}$$ ‖

It is straightforward to prove

Proposition 15 A decentralized equilibrium exists in time periods $t \geq \tau_2$ having the following characteristics: the subset of the population with learning abilities ranging in the interval $A_j \in \left[A_{k-1}, A_J\right]$ acquire privately financed education, whereas the remaining set of individuals with abilities $A_j \in \left[A_1, A_{k-2}\right]$ choose to remain unskilled. The existence of the equilibrium is established upon the condition of occurrence of the following requirements

- The measure of population with ability types $j \in \left[k-1, J\right]$ optimally choose to obtain education.
- The credit market is sustainable.
- Savings be non-negative for all agent types.

Proof

The proof procedure bears an evident analogy to that of *Proposition* 13. We forgo an extensive step-by-step analysis, and plainly assert our argument. *Proposition* 15 is established upon the condition that education be the optimal choice for type *k*-1 agents, as well as the assumption that loan repayment be non-feasible for agent type *k*-2. Hence,

$$A_{k-1}^{\delta} H_t^{1-\delta} > h_{\min} + Rq \qquad \forall t \geq \tau_2 , \tag{119}$$

being reducible in the linear case to

$$A_{k-1} > \frac{h_{\min} + Rq}{H_t} \qquad \forall t \geq \tau_2 . \tag{119'}$$

and

$$A_{k-2}^{\delta} H_t^{1-\delta} < Rq \qquad \forall t \geq \tau_2 \tag{120}$$ ‖

The dynamic evolution of the society's stock of human capital along the time path $t \in\geq \tau_2$ is governed by the first-order difference equation

$$H_{t+1} = h_{\min} \sum_{j=1}^{k-2} \lambda_j + H_t^{1-\delta} \sum_{j=k-1}^{J} \lambda_j A_j^{\delta} \qquad t \in\geq \tau_2 . \tag{121}$$

The solution to the linear form of equation (121) is given by

$$H_t = \begin{cases} h_{\min} \left[(1-\hat{\varphi})\hat{\theta}^t + \hat{\varphi} \right] & \text{if } \hat{\theta} \equiv \sum_{j=k-1}^{J} \lambda_j A_j > 1 \\ & \qquad\qquad \forall t \geq \tau_2 + 1 . \\ h_{\min} \left(1 + t \sum_{j=1}^{k-2} \lambda_j \right) & \text{if } \hat{\theta} \equiv \sum_{j=k-1}^{J} \lambda_j A_j = 1 \end{cases} \tag{122}$$

where we define $\hat{\varphi} \equiv \dfrac{\sum_{j=1}^{k-2} \lambda_j}{1-\hat{\theta}}$.

The economy finally attains a level of per-capita income that defines education the optimal choice for each and every agent type.

Proposition 16 There exists a time period $\tau_n > 0$, where $\tau_n \in [\tau_1, \infty)$, in which the income realization of educated individuals of the lowest ability exceeds the threshold level that defines education the optimal choice. Namely,

$$A_1^{\delta} H_t^{1-\delta} \begin{cases} \geq Rq + h_{\min} & \forall t \in [\tau_n, \infty) \\ < Rq + h_{\min} & \forall t \in [0, \tau_n - 1] \end{cases}. \tag{123}$$

where $A_1 \equiv \min \{A_j\}_{j=1,\dots,J}$. Taking the case of linear technology, τ_n is defined by the following expression

$$\tau_n = \begin{cases} \dfrac{\ln[\overline{\Phi}]}{\ln[\overline{\theta}]}, & if\ \overline{\theta} \equiv \sum_{j=2}^{J} \lambda_j A_j > 1 \\ \dfrac{Rq + h_{\min}(1 - A_1)}{\lambda_1 A_1 h_{\min}}, & if\ \overline{\theta} \equiv \sum_{j=2}^{J} \lambda_j A_j = 1 \end{cases}, \tag{124}$$

where $\overline{\Phi} \equiv \dfrac{Rq + h_{\min}(1 - \overline{\varphi} A_1)}{A_1 h_{\min}(1 - \overline{\varphi})}$, and $\overline{\varphi} \equiv \dfrac{\lambda_1}{1 - \overline{\theta}}$.

Proof

The proof of existence of time period τ_n is exactly analogous to the proof procedure of *Propositions* 12, and 13. Once again, the requirement must be imposed that τ_n be greater to unity. In specific, it must hold true that

- $$\frac{Rq + h_{\min}(1 - \overline{\varphi} A_1)}{A_1 h_{\min}(1 - \overline{\varphi})} > \sum_{j=2}^{J} \lambda_j A_j \qquad if\ \overline{\theta} > 1. \tag{125}$$

- $$A_1 < \frac{Rq + h_{\min}}{(1 + \lambda_1) h_{\min}} \qquad if\ \overline{\theta} = 1. \tag{126}$$ ‖

The proof of existence of the equilibrium path on which growth is supported by human capital investment of each and all types is enclosed in the following theorem

Proposition 17 A competitive equilibrium where the entire population acquires privately financed education exists in each and all time periods of the interval $t \in [\tau_n, \infty)$ on the condition of occurrence of the following requirements

- All types optimally choose to invest in individual improvement.
- The credit market is privately sustainable.
- Individual saving be non-negative for each and all individual types.

Proof

- Each and every individual type must be induced to engage in human capital investment. Once again, this is guaranteed only insofar as

$$A_j^{\delta} H_t^{1-\delta} > h_{\min} + Rq \qquad \forall j \in \{1,\dots,J\}, \forall t \in [\tau_n, \infty). \tag{127}$$

Evidently, it suffices to impose relationship (127) for type $j = 1$ to ensure its validity for all remaining types $j = 2,\dots,J$. Hence,

$$A_1^{\delta} H_t^{1-\delta} > h_{\min} + Rq \qquad \forall t \in [\tau_n, \infty) \tag{127'}$$

Evaluated in the linear case, expression (127') yields

$$A_1 > \frac{h_{\min} + Rq}{H_t} \qquad \forall t \in [\tau_n, \infty). \tag{127''}$$

- In light of Corollary 1, the human capital credit market is sustainable upon the condition that feasibility is established for the lowest-ability agents. The condition translates into

$$A_1^{\delta} H_t^{1-\delta} > Rq \qquad \forall t \in [\tau_n, \infty). \tag{128}$$

- Saving must be positive for all types of agents. Invoking the optimal saving function (equation 8) this means

$$s_{2t+1}^{j*} = (1-\beta)\left(A_j^{\delta} H_t^{1-\delta} - Rq\right) \qquad \forall j \in [1,\dots,J], \forall t \in [\tau_n, \infty), \tag{129}$$

which is strictly positive on the basis of optimality conditions (127). Once again, we need only establish that saving be positive for the lowest-type agents, which evidently is met under the condition (127'). It is trivial to show that *Proposition* 17 is established for the linear case upon the imposition of condition (127''). The validity of the remaining relations is logically implied.$\|$

Our argument on the non-monotonic dynamics of the economy-wide distribution ought not to be qualitatively sensitive to an analysis of higher degree of heterogeneity. Following the same proof procedure, the essence of *Proposition* 7 is here established in the form of a more general argument.

We recall that the income share of the class of type-*j* individuals, at time period *t*, is defined as $sh_t^j \equiv y_t^j / Y_t$, $\forall j \in \{1,\dots,J\}$. The economy-wide income distribution is represented by the set of shares of all income classes $\mathbf{sh}_{t+1}^{\ell} = \{sh_{t+1}^{1\,\ell},\dots,sh_{t+1}^{J\,\ell}\}$, for $t \geq 0$, with ℓ denoting the stage of equilibrium growth, $\ell \in \{I, II,\dots,J-k+1\}$. We now proceed to establish that the economy-wide income distribution in the state of poverty Lorenz dominates the distribution of the equilibrium where the subset of the population with abilities ranging in the interval $j \in [k+1, J]$ invests in human capital accumulation.

Invoking the aforementioned definition of income share, as well as the equations on individual and average income (22) and (23) respectively, we obtain the following expression for the share of income classes in the poverty equilibrium:

$$sh_{t+1}^{j\,I} = 1 \qquad j \in [1, J], \forall t \in [0, r-1]. \tag{130}$$

As has been previously stated, in the phase of underdevelopment genetic differences in learning aptitude *vanish* in the sense that they are not reflected in the income earning ability of agents. This is a state of perfect income equality, with all agents earning the minimum average income, $h_{\min}$.

We recall that adaptations in the legislative system to accommodate *strong* legal protection of creditors are effective on period $t = r$. Along the growth path following such development the various types of agents earn the respective income shares

$$sh_{t+1}^{j\,II} = h_{\min} \Big/ \left\{ h_{\min} \sum_{j=1}^{k} \lambda_j + H_t^{II\,1-\delta} \sum_{j=k+1}^{J} \lambda_j A_j^{\delta} \right\} \qquad j \in [1, k], \forall t \in [r, \tau_1 - 1]. \tag{131}$$

$$sh_{t+1}^{j\,II} = A_j^{\delta} H_t^{II\,1-\delta} \Big/ \left\{ h_{\min} \sum_{j=1}^{k} \lambda_j + H_t^{II\,1-\delta} \sum_{j=k+1}^{J} \lambda_j A_j^{\delta} \right\} \qquad j \in [k+1, J], \forall t \in [r, \tau_1 - 1]. \tag{132}$$

Applying the criterion of Lorenz superiority, we obtain that the income distribution $\mathbf{sh}_{t+1}^{I}$ Lorenz dominates distribution $\mathbf{sh}_{t+1}^{II}$ under the condition that the following requirements be met

- $\lambda_1 sh_{t+1}^{1\,II} \le \lambda_1 sh_{t+1}^{1\,I} \qquad \forall t \in [r, \tau - 1],$ (133)

implying

$$h_{\min} \sum_{j=k+1}^{J} \lambda_j \le H_t^{II\,1-\delta} \sum_{j=k+1}^{J} \lambda_j A_j^{\delta} \qquad \forall t \in [r, \tau_1 - 1]. \tag{133'}$$

- $\lambda_1 sh_{t+1}^{1\,II} + \lambda_2 sh_{t+1}^{2\,II} \le \lambda_1 sh_{t+1}^{1\,I} + \lambda_2 sh_{t+1}^{2\,I} \qquad \forall t \in [r, \tau_1 - 1],$ (134)

which also reduces to relation (133').

- $\lambda_1 sh_{t+1}^{1\,II} + \lambda_2 sh_{t+1}^{2\,II} + \ldots + \lambda_k sh_{t+1}^{k\,II} \le \lambda_1 sh_{t+1}^{1\,I} + \lambda_2 sh_{t+1}^{2\,I} + \ldots + \lambda_k sh_{t+1}^{k\,I} \qquad \forall t \in [r, \tau_1 - 1],$ (135)

similarly being true on the basis of relation (133').

- $\lambda_1 sh_{t+1}^{1\,II} + \ldots + \lambda_k sh_{t+1}^{k\,II} + \lambda_{k+1} sh_{t+1}^{k+1\,II} \le \lambda_1 sh_{t+1}^{1\,I} + \ldots + \lambda_k sh_{t+1}^{k\,I} + \lambda_{k+1} sh_{t+1}^{k+1\,I} \qquad \forall t \in [r, \tau_1 - 1],$ (136)

which reads into

$$h_{\min} \sum_{j=1}^{k} \lambda_j + \lambda_{k+1} A_{k+1}^{\delta} H_t^{II\,1-\delta} \le \sum_{j=1}^{k+1} \lambda_j \left\{ h_{\min} \sum_{j=1}^{k} \lambda_j + H_t^{II\,1-\delta} \sum_{j=k+1}^{J} \lambda_j A_j^{\delta} \right\} \qquad \forall t \in [r, \tau_1 - 1]. \tag{136'}$$

- $\lambda_1 sh_{t+1}^{1\,II} + \ldots + \lambda_k sh_{t+1}^{k\,II} + \ldots + \lambda_J sh_{t+1}^{J\,II} \le \lambda_1 sh_{t+1}^{1\,I} + \ldots + \lambda_k sh_{t+1}^{k\,I} + \ldots + \lambda_J sh_{t+1}^{J\,I} \qquad \forall t \in [r, \tau_1 - 1].$ (137)

whose validation can be much too easily proved (each side equals unity).

In the last stage of development, $\bar{\ell} \equiv J - k + 1$, educational investment is consistent with optimal incentives for the array of all types of individuals. Along this equilibrium state, the various classes of agents earn the following share of aggregate output

$$sh_{t+1}^{j\,\bar{\ell}} = A_j^{\delta} \Big/ \sum_{j=1}^{J} \lambda_j A_j^{\delta} \qquad j \in [1, J], \forall t \in [\tau_n, \infty). \tag{138}$$

We establish that distribution $\mathbf{sh}_{t+1}^{II}$ is characterized by greater income inequality in terms of Lorenz superiority compared to the income distribution of the last phase of development, $\mathbf{sh}_{t+1}^{\bar{\ell}}$. As it is known, the proof entails the following requirements

- $\lambda_1 sh_{t+1}^{1\,II} \le \lambda_1 sh_{t+1}^{1\,\bar{\ell}} \qquad \forall t \in [\tau_n, \infty),$ (139)

which is satisfied upon the truth of the following relation

$$h_{\min} \sum_{j=1}^{J} \lambda_j A_j^{\delta} \le A_1^{\delta} \left\{ h_{\min} \sum_{j=1}^{k} \lambda_j + H_t^{II\,1-\delta} \sum_{j=k+1}^{J} \lambda_j A_j^{\delta} \right\} \qquad \forall t \in [\tau_n, \infty). \tag{139'}$$

It must further be met

- $\lambda_1 sh_{t+1}^{1\,II} + \lambda_2 sh_{t+1}^{2\,II} \le \lambda_1 sh_{t+1}^{1\,\bar{\ell}} + \lambda_2 sh_{t+1}^{2\,\bar{\ell}} \qquad \forall t \in [\tau_n, \infty),$ (140)

being validated on the basis of the condition

$$(\lambda_1 + \lambda_2) h_{\min} \sum_{j=1}^{J} \lambda_j A_j^{\delta} \le \left(\lambda_1 A_1^{\delta} + \lambda_2 A_2^{\delta}\right) \left\{ h_{\min} \sum_{j=1}^{k} \lambda_j + H_t^{II\,1-\delta} \sum_{j=k+1}^{J} \lambda_j A_j^{\delta} \right\} \qquad \forall t \in [\tau_n, \infty). \tag{140'}$$

- $\lambda_1 sh_{t+1}^{1\,II} + \lambda_2 sh_{t+1}^{2\,II} + \ldots + \lambda_k sh_{t+1}^{k\,II} \le \lambda_1 sh_{t+1}^{1\,\bar{\ell}} + \lambda_2 sh_{t+1}^{2\,\bar{\ell}} + \ldots + \lambda_k sh_{t+1}^{k\,\bar{\ell}} \qquad \forall t \in [\tau_n, \infty),$ (141)

being equivalent to

$$h_{\min} \sum_{j=1}^{k} \lambda_j \sum_{j=1}^{J} \lambda_j A_j^{\delta} \le \sum_{j=1}^{k} \lambda_j A_j^{\delta} \left\{ h_{\min} \sum_{j=1}^{k} \lambda_j + H_t^{II\,1-\delta} \sum_{j=k+1}^{J} \lambda_j A_j^{\delta} \right\} \qquad \forall t \in [\tau_n, \infty). \tag{141'}$$

- $\lambda_1 sh_{t+1}^{1\,II} + \ldots + \lambda_k sh_{t+1}^{k\,II} + \lambda_{k+1} sh_{t+1}^{k+1\,II} \leq \lambda_1 sh_{t+1}^{1\,\bar{\ell}} + \ldots + \lambda_k sh_{t+1}^{k\,\bar{\ell}} + \lambda_{k+1} sh_{t+1}^{k+1\,\bar{\ell}} \qquad \forall t \in [\tau_n, \infty),$ (142)

implying

$$\left\{ h_{\min} \sum_{j=1}^{k} \lambda_j + \lambda_{k+1} A_{k+1}^{\delta} H_t^{II\,1-\delta} \right\} \sum_{j=1}^{J} \lambda_j A_j^{\delta} \leq \sum_{j=1}^{k+1} \lambda_j A_j^{\delta} \left\{ h_{\min} \sum_{j=1}^{k} \lambda_j + H_t^{II\,1-\delta} \sum_{j=k+1}^{J} \lambda_j A_j^{\delta} \right\}, \ \forall t \in [\tau_n, \infty). \qquad (142')$$

Lastly,

- $\lambda_1 sh_{t+1}^{1\,II} + \ldots + \lambda_k sh_{t+1}^{k\,II} + \ldots + \lambda_{J-1} sh_{t+1}^{J-1\,II} \leq \lambda_1 sh_{t+1}^{1\,\bar{\ell}} + \ldots + \lambda_k sh_{t+1}^{k\,\bar{\ell}} + \ldots + \lambda_{J-1} sh_{t+1}^{J\,\bar{\ell}} \qquad \forall t \in [\tau_n, \infty),$ (143)

which yields

$$\left\{ h_{\min} \sum_{j=1}^{k} \lambda_j + H_t^{II\,1-\delta} \sum_{j=k+1}^{J-1} \lambda_j A_j^{\delta} \right\} \sum_{j=1}^{J} \lambda_j A_j^{\delta} \leq \sum_{j=1}^{J-1} \lambda_j A_j^{\delta} \left\{ h_{\min} \sum_{j=1}^{k} \lambda_j + H_t^{II\,1-\delta} \sum_{j=k+1}^{J} \lambda_j A_j^{\delta} \right\} \quad \forall t \in [\tau_n, \infty). \qquad (143')$$

The reader may find the *Kuznets curve* of this version of the model in the Appendix.

VII. *Less stringent punishment scheme*

In this section we pursue an extension of the model in which individual consumption in old age is expanded to include a fixed retirement income. By assumption, all economic agents receive the same real endowment, irrespective of one's educational status. The objective underlying this approach is to determine the *ability* of the economy to sustain the existence of a private credit market in the education area when the punishment scheme is effectively weakened. This is examined by actually rendering default less costly (whereas previously $V^{E,D}(j) \to -\infty$, in the present case $V^{E,D}(j) > 0$). We show that a market for education loans may be privately sustained in this context, albeit at the cost of stricter conditions.

The budget constraint in the last period of life is modified to be

$$c_{3t+2}^{j} = \omega_3 + R s_{2t+1}^{j}, \qquad (144)$$

where ω_3 denotes the endowment in real units. Adopting the utility function of logarithmic form (7), optimal saving is now expressed as

$$s_{2t+1}^{j*} = \begin{cases} \dfrac{(1-\beta)R\left(A_j^{\delta} H_t^{1-\delta} - Rq\right) - \beta\omega_3}{R} & if \ q > 0 \\ \dfrac{(1-\beta)R\,h_{\min} - \beta\omega_3}{R} & if \ q = 0 \end{cases} \quad \forall j = L, H. \qquad (145)$$

Upon substitution of the saving function (145) into the budget constraints (5") and (144), we obtain the optimal second- and third-period consumption, respectively given by

$$c_{2t+2}^{j*} = \begin{cases} \dfrac{\beta\left\{R\left(A_j^{\delta} H_t^{1-\delta} - R\,q\right) + \omega_3\right\}}{R} & if\;\; q > 0 \\ \dfrac{\beta\left\{R\,h_{\min} + \omega_3\right\}}{R} & if\;\; q = 0 \end{cases} \quad \forall j \in \{L, H\}. \tag{146}$$

$$c_{3t+2}^{j*} = \begin{cases} (1-\beta)\left\{R\left(A_j^{\delta} H_t^{1-\delta} - R\,q\right) + \omega_3\right\} & if\;\; q > 0 \\ (1-\beta)\left(R\,h_{\min} + \omega_3\right) & if\;\; q = 0 \end{cases} \quad \forall j \in \{L, H\}. \tag{147}$$

State of underdevelopment Given that a member of generation *t* receives education her intertemporal consumption if she chooses to default is described by equations

$$\left(c_{2t+1}^{j*}\right)^{WR,D} = \frac{\beta\left(R\,A_j^{\delta} H_t^{1-\delta} + \omega_3\right)}{R} \qquad \forall j \in \{L, H\}, \tag{148}$$

and

$$\left(c_{3t+2}^{j*}\right)^{WR,D} = (1-\beta)\left(R\,A_j^{\delta} H_t^{1-\delta} + \omega_3\right) \qquad \forall j \in \{L, H\}, \tag{149}$$

When remaining loyal to contract commitment optimal adult- and old-age consumption is given by, respectively

$$\left(c_{2t+1}^{j*}\right)^{ND} = \frac{\beta\left\{R\left(A_j^{\delta} H_t^{1-\delta} - R\,q\right) + \omega_3\right\}}{R} \quad q > 0,\ \forall j \in \{L, H\}, \tag{150}$$

and

$$\left(c_{3t+2}^{j*}\right)^{ND} = (1-\beta)\left\{R\left(A_j^{\delta} H_t^{1-\delta} - Rq\right) + \omega_3\right\} \qquad q > 0, \forall j \in \{L, H\}, \tag{151}$$

where $\left(c_{\nu t+1}^{j*}\right)^{WR,ND} = \left(c_{\nu t+1}^{j*}\right)^{SR,ND} \equiv \left(c_{\nu t+1}^{j*}\right)^{ND}$ for *v* = 2, 3. Evidently, utility is higher when evading debt obligations due to higher second-period consumption, and because agents can still engage in intertemporal smoothing through saving. Hence,

$$V_t^{WR,D}(j) > V_t^{ND}(j) \qquad q > 0, \forall j \in \{L, H\}, \tag{152}$$

where

$$V_t^{WR,D}(j) = \ln\left\{\vartheta\left(R\,A_j^{\delta} H_t^{1-\delta} + \omega_3\right)\right\} \qquad \forall j \in \{L, H\}, \tag{153}$$

and

$$V_t^{E,ND}(j) = \ln\left\{\vartheta\left(R\left(A_j^{\delta}\, H_t^{1-\delta} - Rq\right) + \omega_3\right)\right\} \qquad q > 0\,, \forall j \in \{L, H\}, \tag{154}$$

with $\vartheta \equiv \beta^{\beta}(1-\beta)^{1-\beta}/R^{\beta}$. Obviously, it applies $V_t^{WR,ND}(j) = V_t^{SR,ND}(j) \equiv V_t^{E,ND}(j)\ \ \forall j = L, H$. The conclusion is reached that an individual who acquires education shall always commit default on her debt. Were one to receive no education she would earn the unskilled income $h_{\min}$, and hence lifetime utility

$$V_t^{NE}(j) \equiv V_t^{NE} = \ln\left[\vartheta\left(R\,h_{\min} + \omega_3\right)\right] \quad \forall j \in \{L, H\}. \tag{155}$$

Drawing upon *Assumption* 1, we infer that remaining unskilled is never the preferred choice. It is evident that

$$V_t^{WR,D}(j) > V_t^{NE} \qquad \forall j \in \{L, H\} \tag{156}$$

Similarly to the core version (where $\omega_3 = 0$) the model predicts that financial institutions engage in no educational funding as a result of the expected lack of commitment on part of borrowers. Once again, owing to the rationing of all credit, the entire population remains uneducated earning the minimum income of unskilled labor. On the assumption that a system of weak legal rights prevails in each period of the time interval $t \in [0, r-1]$, $r > 0$, the equilibrium path has the characteristics of a poverty trap

$$y_{t+1}^{j} = h_{t+1}^{j} = h_{\min} \quad \forall j \in \{L, H\}, \forall t \in [0, r-1]. \tag{157}$$

The competitive outcome along this equilibrium path prescribes that the economy produces the time-invariant quantity

$$Y_{t+1}^{NE} = H_{t+1}^{NE} = h_{\min}, \tag{158}$$

where we recall $y_0^{j} \equiv y_0 = h_{\min}$, $\forall\, j$, thus $Y_0 = h_{\min}$.

A segment of the society invests Once again, we take as our basis that effective on period $t = r$, $r > 1$, legislation entitles creditors to seize the entire assets of a debtor in default, effectively prohibiting the latter from any act of saving as time unfolds.

The contract design elicits promise-keeping behavior for the high-type agent only insofar as

$$V_t^{E,ND}(j) > V_t^{SR,D}(j) \qquad \textit{for } j = H\,, \forall t \geq r+1. \tag{159}$$

The present utility value associated with repudiating on one's debt is given by

$$V_t^{SR,D}(j) = \ln\left\{\left(A_j^{\delta}\, H_t^{1-\delta}\right)^{\beta} \omega_3^{1-\beta}\right\} \qquad \forall j = L, H\,, \forall t \geq r+1. \tag{160}$$

Drawing upon equation (154), there results that the following condition must be imposed

$$\vartheta\left\{R\left(A_H^{\delta} H_t^{1-\delta} - Rq\right) + \omega_3\right\} > \left(A_H^{\delta} H_t^{1-\delta}\right)^{\beta} \omega_3^{1-\beta} \qquad \forall t \geq r+1. \tag{159'}$$

Taking as our basis *Assumption* 2, we note again that relation (159) cannot possibly hold for the low-type agents, since the logarithmic function $V^{E,ND}(j)$ is non-definable on a negative argument. Insofar as the only possibility is to renege on the agreement, the household attains the utility level associated with no consumption smoothing. $V^{E,ND}(L)$ in effect degenerates to individual welfare $V^{SR,D}(L)$. We quote the proposition

Proposition 18 The contract arrangement (q, R) offered in a system where borrowers have no access to savings opportunities conditional on default may be supported as a self-enforcing contract. Granting the feasibility of loan repayment, if consumption in old age falls not below the fixed threshold ω_3, constraint (159') is required in order for borrowers to renege not on agreed obligations.

The following theorem is proved without difficulty

Proposition 19 A private credit market for human capital investment is *sustainable* if and only if

- Given the feasibility of loan repayment agents seeking credit are offered a self-enforcing contract.
- Individuals for whom debt repayment is non-feasible prefer to receive no education.

Proof

- We have established that upon the validity of relationship (159'), high-ability agents optimally choose to adhere to the contract agreement.
- The second necessary condition requires that low ability agents optimally choose to remain unskilled. On the basis of a plausible condition, it holds that low-type agents indeed prefer to earn the low income of unskilled labor, while maintaining their ability to enhance consumption in the retirement age beyond the fixed endowment ω_3. The optimality condition states:

$$V_t^{NE} > V_t^{E,D}(L) \qquad \forall t \geq r+1. \tag{161}$$

Upon invoking equations (155) and (160), we obtain that the following hypothesis must be imposed:

$$\vartheta\left(Rh_{\min} + \omega_3\right) > \left(A_L^{\delta} H_t^{1-\delta}\right)^{\beta} \omega_3^{1-\beta} \qquad \forall t \geq r+1. \tag{161'}$$ ‖

We conclude this section with the following proposition:

Proposition 20 A competitive equilibrium with a subset of population acquiring privately financed education exists on the condition of occurrence of the following requirements

- The measure of high-type agents optimally chooses to obtain education.
- The credit market is privately sustainable.
- Individual savings is non-negative for both H and L types.

Proof

- The measure of high-type agents is induced to participate in the contract arrangement if the following condition is met

$$V_t^{E,ND}(H) > V_t^{NE} \qquad \forall t \geq r . \tag{162}$$

which, upon invoking equations (154) and (155), translates into

$$A_H^{\delta} H_t^{1-\delta} > Rq + h_{\min} \qquad \forall t \geq r . \tag{162'}$$

Evaluated in period $t = r$, we have

$$A_H^{\delta} h_{\min}^{1-\delta} > Rq + h_{\min} . \tag{162''}$$

Evidently, it suffices to impose relationship (162'') to ensure the validity of (162') in all forthcoming periods, $t > r$.

- In light of *Proposition* 19, the human capital credit market is privately sustainable upon the validity of *Assumption* 2, as well as conditions (159') and (161').
- The individual rationality constraint for financial entities entails that saving be positive for both types of individuals, which reads into the conditions

$$A_H^{\delta} H_t^{1-\delta} \geq Rq + \frac{\beta \omega_3}{(1-\beta)R} \qquad \forall t \geq r . \tag{163}$$

and

$$h_{\min} \geq \frac{\beta \omega_3}{(1-\beta)R} \qquad \forall t \geq r . \tag{164}$$

The proof of *Proposition* 20 consists of imposing *Assumption* 2(β), and relations (159'), (161'), (162'') and (164). The truth of the remaining conditions is then logically inferred.‖

The dynamic evolution of the society's stock of human capital along the aforementioned equilibrium path is governed by the first-order non-linear difference equation (30). Similarly to the baseline model, the solution to the linear version is given by the expressions (31).

All types invest Due to perpetual growth the stock of aggregate knowledge reaches a critical threshold, above which individuals of both types have an optimal incentive to invest. Similarly to the core version of the model, we establish this proposition for the case of the linear human capital technology. A restatement of *Proposition* 4 reads as follows

Proposition 21 There exists a time period $\hat{\tau} > 0$, where $\hat{\tau} \in [r+1, \infty)$, in which the income realization of educated low-type agents exceeds the threshold level that defines education the optimal choice. Mathematically, this reads into

$$A_L^{\delta} H_t^{1-\delta} \begin{cases} \geq Rq + h_{\min} & \forall t \in [\hat{\tau}, \infty) \\ < Rq + h_{\min} & \forall t \in [0, \hat{\tau}-1] \end{cases}. \qquad (165)$$

Considering the linear human capital technology, $\hat{\tau}$ is defined as

$$\hat{\tau} = \begin{cases} \dfrac{\ln[\Theta]}{\ln[\lambda A_H]}, & if\ \lambda A_H > 1 \\ \dfrac{Rq + h_{\min}(1 - A_L)}{A_L h_{\min}(1-\lambda)}, & if\ \lambda A_H = 1 \end{cases}, \qquad (166)$$

with $\Theta \equiv \dfrac{(Rq + h_{\min})(1 - \lambda A_H) - A_L h_{\min}(1-\lambda)}{A_L h_{\min} \lambda (1 - A_H)}$.

The condition that defines education the optimal choice (relation 162), is written as

$$A_L^{\delta} H_t^{1-\delta} \geq Rq + h_{\min}. \qquad (165')$$

This is identical to the corresponding condition in the baseline version of the model $(\omega_3 = 0)$. It is only evident that the two versions imply identical solutions for the critical time period τ, hence $\tau = \hat{\tau}$. The proof of the theorem being identical to that of *Proposition* 4 is here omitted.

Proposition 1 is rephrased in our context to read as follows

Proposition 22 The contract arrangement (*q*, *R*) offered in a system where borrowers have no access to savings opportunities conditional on default, may be supported as a self-enforcing contract. Granting the feasibility of loan repayment for all types of agents, conditions (167') ought to be imposed in order for borrowers to renege not on agreed obligations.

Proof

The feasibility of loan repayment is ensured for both types of agents under the fulfillment of relation (165'). It goes without saying, then, that $V^{E,ND}(j)$ is a definable function, and positive for both *H* and *L* types (see equation 154). The contract arrangement (*q*, *R*) is self-enforcing for agents of both ability levels on condition that relation (159) applies for $\forall j \in \{L, H\}$. Evidently, this reads into

$$\vartheta\left\{R\left(A_L^{\delta} H_t^{1-\delta} - Rq\right) + \omega_3\right\} > \left(A_L^{\delta} H_t^{1-\delta}\right)^{\beta} \omega_3^{1-\beta} \qquad \forall t \geq r+1, \tag{167α}$$

$$\vartheta\left\{R\left(A_H^{\delta} H_t^{1-\delta} - Rq\right) + \omega_3\right\} > \left(A_H^{\delta} H_t^{1-\delta}\right)^{\beta} \omega_3^{1-\beta} \qquad \forall t \geq r+1, \tag{167β}$$

The theorem logically follows:

Proposition 23 The credit market for educational investment is privately *sustainable* in each and all time periods $t \in [\hat{\tau}, \infty)$ given the validity of condition (165'), and optimality constraints (167).

The proof of existence of the equilibrium in which the entire population invests is enclosed in the following proposition

Proposition 24 A competitive equilibrium where the entire population acquires privately financed education exists on the condition of occurrence of the following requirements

- Both types make the optimal decision to invest in individual improvement.
- The credit market is privately sustainable.
- Individual saving is non-negative for both *H* and *L* types.

Proof

- The *participation constraint* for the borrower side entails that condition (162) applies for both types of agents. This translates into

$$A_j^{\delta} H_t^{1-\delta} > Rq + h_{\min} \qquad \forall j = L, H, \tag{168}$$

which is reducible to the optimality condition (165').

- Drawing on *Proposition* 24, we assert that the financial market for human capital investment is privately sustainable upon the validity of optimality conditions (165'), and (167*α*, *β*).
- Loan provision ought to be individually rational for credit institutions, which implies the conditions

$$s_{2t+1}^{j*} = (1-\beta)\left(A_j^{\delta} H_t^{1-\delta} - Rq\right) \qquad \forall j \in \{L, H\}, \forall t \in [\hat{\tau}, \infty). \tag{169}$$

It is evident that individual saving is strictly positive for both types on the basis of optimality condition (165'). In summary, the proof of this proposition requires the validity of optimality conditions (165'), and (167α, β).‖

As in the core version of the model, the dynamic evolution of the economy's aggregate stock of knowledge is governed by the non-linear difference equation (42). The solution to the linear case is given by equations (43). An analysis on income distribution is omitted here due to being identical to that of the benchmark version (see Section IV).

VIII. *Concluding remarks*

Let us cast a glance backward on the course of this essay. We sought to construct a theory which in a novel way lends truth to the proposition formed by Kuznets (1955), with respect to the non-monotonic relationship between prosperity and inequality of income distribution. Our attention centered on the role of financial markets in defining the process of economic development, and ultimately the distribution of income earning capabilities in a population of *ex ante* heterogeneous individuals. If the roots of development lie in human capital accumulation, the possibility to fund educational choices through private credit organizations is critical in its own right. The theory abstracts from the possibility of education be publicly provided, and of alternative means of financing human capital investment, through wealth possessions or forms of inherited bequests. Owing to this confinement it is a consequence of the failure of the credit market that individuals may be entirely barred from productive educational choices. In this circumstance, the potential for differing earning productivities remains unrealized, with all workers being *trapped* in the choice of a single low-income occupation, and therefore identical earnings. The provision of credit in this market is hindered by one-sided lack of commitment, and particular enforcement issues embedded in the area of educational investment. Contract enforcement hinging on the nature of consequences following an act of default ultimately is a matter of the legislative system. In the tradition of Kehoe and Levine (1993) we assume that legislation accommodates the complete and permanent exclusion of defaulting borrowers from financial markets. The prospect of being prohibited to invest in tangible assets induces agents to choose commitment to previous agreements. Contract arrangements thus become enforceable, leading credit institutions to eagerly engage in educational funding. This is the critical requirement for the economy to be carried on a dynamic path of ever sustained growth, escaping a poverty loophole. We trace out paths of development so constructed as to give an explicit proof of the *trickle-down* theory of economic growth. Initially, an equilibrium is taken to exist in which a particular group of individuals, those with the highest investment return, only choose to engage in education. Owing to the accumulation of human capital and the associated externality on future generations' knowledge productivity, the economy ultimately makes its transition to a state where the aggregate of all agents invest in individual improvement. As endogenous technological knowledge takes off, the externality effect arising from knowledge spillovers gives rise to inverted-*U* dynamics in the evolution of income distribution. A pattern of worsening inequality prevails in early stages of growth. However, as dynamics bring the economy on a more evolved stage, income differentials appear to shrink. Income convergence is established to be the signal that an advanced level of development has been attained.

Following the mainstream tradition, our enquiry on the dynamic pattern of wage income distribution was accommodated in the context of a deterministic theory. Implicit in our model is the assumption that no unpredictable change ever occurs, or at least economic agents believe that it doesn't. Consequently, the study remains silent about the impact of uncertainty on the actions of individual agents, and thus on the dynamics of the system. A model denying that economic variables may be inherently unpredictable leaves valid questions quite unanswered, thus weakening the position of proven theorems. An endeavor sought in future work is to extend the existing analysis in a way that uncertainty is embedded into investment decisions in human capital.

- Adelman, I. and C. T. Morris (1973) *Economic Growth and Social Equity in Developing Countries*, Stanford University Press, Stanford, CA.
- Adelman, I. and Robinson, S. (1989) *Income Distribution and Development* in Handbook of Development Economics, vol. II (ed. H. Chenery and T. N. Srinivasan), pp.949-1003. Amsterdam: North-Holland.
- Aghion, P. and P. Bolton (1997) “A Theory of Trickle-Down Growth and Development” *Review of Economic Studies* vol.64 pp.151-172.
- Aghion, P. and P. Howitt (1997) *Endogenous Economic Growth* Cambridge MA: MIT Press.
- Ahluwalia, M. S. (1976*a*) “Income Distribution and Development: Some Stylized Facts” *American Economic Review* vol.66 pp.128-135.
- Ahluwalia, M. S. (1976*b*) “Inequality, Poverty and Development” *Journal of Development Economics* vol.3 pp.307-342.
- Ahluwalia, M. S., N. G. Carter and H. B. Chenery (1979) “Growth and Poverty in Developing Countries” *Journal of Development Economics* 6 pp.299-341.
- Alvarez, F. and U. J. Jermann (2000) “Efficiency, Equilibrium, and Asset Pricing with Risk of Default” *Econometrica* vol.68 pp.775-797.
- Anand, S. and S. M. R. Kanbur (1993) “The Kuznets Process and the Inequality-Development Relationship” *Journal of Development Economics* vol.40 pp.25-52.
- Andolfatto, D. and M. Gervais (2006) “Human Capital Investment and Debt Constraints” *Review of Economic Dynamics* vol.9 pp.52-67.
- Azariadis, C. and A. Drazen (1990) “Threshold Externalities in Economic Development” *Quarterly Journal of Economics* vol.105, pp.501-525.
- Azariadis, C. and L. Lambertini (2003) “Endogenous Debt Constraints in Lifecycle Economies” *Review of Economic Studies* vol.70 pp.461-487.
- Bacha, E. L. (1977) *The Kuznets Curve and Beyond: Growth and Changes in inequalities* Development Discussion Papers no.29, Harvard Institute for International Development, Harvard University, Cambridge, MA.

- Bacha, E. L. (1979) "Notes on the Brazilian Experience with Mini-devaluations, 1968-1976" *Journal of Development Economics* vol.6 pp.463-481.
- Banerjee, A. and A. Newman (1993) "Occupational Choice and the Process of Development" *Journal of Political Economy* vol.101, pp.274-299.
- Barro, R. J. (2000) "Inequality and Growth in a Panel of Countries" *Journal of Economic Growth* vol.5, pp.5-32.
- Barro, R. J., N. G. Mankiw and X. Sala-i-Martin (1995) "Capital Mobility in Neoclassical Models of Growth" *American Economic Review* vol.85 pp.103-115.
- Becker, G. (1964) *Human Capital*, New York: NBER and Columbia University Press.
- Becker, G. S. and N. Tomes (1986) "Human Capital and the Rise and Fall of Families" *Journal of Labor Economics* vol.4 pp.S1-S39.
- Benabou, R. (1996) "Heterogeneity, Stratification, and Growth: Macroeconomic Implications of Community Structure and School Finance" *American Economic Review* vol. 86 pp. 584-609.
- Benabou, R. (2002) "Tax and Education Policy in a Heterogeneous-Agent Economy: What Levels of Redistribution Maximize Growth and Efficiency" *Econometrica* vol. 70 pp. 481-517.
- Bencivenga, V. R., and B. D. Smith (1991) "Financial Intermediation and Endogenous Growth" *Review of Economic Studies* vol.58 pp.195-209.
- Behrman, J. R., R. A. Pollak, and P. Taubman (1989) "Family Resources, Family Size, and Access to Financing for College Education" *Journal of Political Economy* vol.97 pp.398-419.
- Benhabib, J. and R. Perli (1994) "Uniqueness and Indeterminacy: On the Dynamics of Endogenous Growth" *Journal of Economic Theory* vol.63 pp.113-142.
- Bond, P. and A. Krishnamurthy (2004) "Regulating Exclusion from Financial Markets" *Review of Economic Studies* vol.71 pp.681-707.
- Bourguignon, F. and C. Morrison (1990) "Income Distribution, Development and Foreign Trade: A Cross Sectional Analysis" *European Economic Review* vol.34 pp.1113-1132.
- Bovenberg, A. L., van Ewijk, C. (1997) "Progressive Taxes, Equity and Human Capital Accumulation in an Endogenous Growth Model with Overlapping Generations" *Journal of Public Economics* vol.64 pp.153-179.

- Buiter, W. and K. Kletzer (1992) "Permanent International Productivity Growth Differentials in an Integrated Global Economy", NBER Working Paper no.4220.
- Chamley, C. (1993) "Externalities and Dynamics in Models of Learning or Doing" *International Economic Review* vol.34 pp.583-609.
- Chatterjee, S. and B. Ravikumar (1999) "Minimum Consumption Requirements: Theoretical and Quantitative Implications for Growth and Distribution" *Macroeconomic Dynamics* vol.3 pp.482-505.
- Chenery, H. B., M. S. Ahluwalia, C. L. G. Bell, J. H. Duloy and R. Jolly (1974) "Redistribution with Growth: An Approach to Policy" Oxford University Press, Oxford.
- Coleman, J. S. *et al*. (1966) *Equality of Educational Opportunity* Washington, D.C. US. GPO.
- De Gregorio, J. (1996) "Borrowing Constraints, Human Capital Accumulation, and Growth" *Journal of Monetary Economics* vol.37 pp.49-71.
- De Gregorio, J. and S. Kim (2000) "Credit Markets with Differences in Abilities: Education, Distribution, and Growth" *International Economic Review* vol.41 pp.579-607.
- De la Croix, D. and P. Michael (2007) "Education and Growth with Endogenous Debt Constraints" *Economic Theory* vol.33 pp.509-530.
- Diamond, P. A. (1965) "National Debt in a Neoclassical Growth Model" *American Economic Review* vol.55 pp.1126-1150.
- Djankov, S., C. McLiesh and A. Shleifer (2007) "Private Credit in 129 Countries" *Journal of Financial Economics* vol.84 pp.299-329.
- Eaton, J. and M. Gersovitz (1981) "Debt with Potential Repudiation: Theoretical and Empirical Analysis" *Review of Economic Studies* vol.48 pp.289-309.
- Eaton, J. and H. S. Rosen (1980) "Taxation, Human Capital, and Uncertainty" *The American Economic Review* vol. 70 pp.705-715.
- Fender, J. and P. Wang (2003) "Educational Policy in a Credit Constrained Economy with Skill Heterogeneity" *International Economic Review* vol.44 pp.939-964.
- Friedman, M. (1962) *Capitalism and Freedom* University of Chicago Press, Chicago, IL.

- Galor, O. and D. Tsiddon (1997a) "Technological Progress, Mobility, and Economic Growth" *American Economic Review* vol.87 pp.363-382.
- Galor, O. and D. Tsiddon (1997b) "The Distribution of Human Capital and Economic Growth" *Journal of Economic Growth* vol.2 pp.93-124.
- Galor, O. and J. Zeira (1993) "Income Distribution and Macroeconomics" *Review of Economic Studies* vol.60 pp.35-52.
- Goldsmith, R. W. (1969) *Financial Structure and Development*. New Haven, Conn.: Yale Univ. Press.
- Green, E. J. (1987) "Lending and the Smoothing of Uninsurable Income." In Edward C. Prescott and Neil Wallace (eds.), *Contractual Arrangements for Intertemporal Trade, Minnesota Studies in Macroeconomics Series, Vol. 1*. Minneapolis: University of Minnesota Press, pp.3-25.
- Greenwood, J. and B. Jovanovic (1990) "Financial Development, Growth, and the Distribution of Income" *Journal of Political Economy* vol.98 pp.1076-1107.
- Griliches, Z. and W. M. Mason (1972) "Education, Income and Ability" *Journal of Political Economy* vol.80(3) pp.S74-S103.
- Helpman, E. (1997) *General Purpose Technologies and Economic Growth* Cambridge, MA: MIT Press.
- Hendricks, L. (1999) "Taxation and Long-run Growth" *Journal of Monetary Economics* vol.43 pp.411-434.
- Jaffee, D. and T. Russell (1976) "Imperfect Information, Uncertainty, and Credit Rationing" *Quarterly Journal of Economics* vol.90 pp.651-666.
- Jaffee, D. and J. Stiglitz (1990) "Credit Rationing" in B. Friedman and F. Hahn, *Handbook of Monetary Economics* vol.2 (Amsterdam: North-Holland).
- Jappelli, T. and M. Pagano (2002) "Information Sharing, Lending, and Defaults: Cross-country Evidence" *Journal of Banking and Finance* vol.26 pp.2017-2045.
- Kehoe, T. J. and D. K. Levine (1993) "Debt-Constrained Asset Markets" *Review of Economic Studies* vol.60 pp.865-888.
- King, R. G. and R. Levine (1993) "Finance Entrepreneurship and Growth" *Journal of Monetary Economics* vol.32 pp.513-542.
- Kuznets, S. (1955) "Growth and Income Inequality" *American Economic Review* vol.45 pp.1-28.

- La Porta, R., F. Lopez-de-Silanes, A. Shleifer and R.W. Vishny (1998) "Law and Finance" *Journal of Political Economy* vol.106 pp.1113-1155.
- Laing, D., T. Palivos and P. Wang (2003) "The Economics of 'New Blood" *Journal of Economic Theory* vol. 112 p.106-156.
- Levhari, D. and Y. Weiss (1974) "The Effect of Risk on the Investment in Human Capital" *The American Economic Review* vol. 64 pp. 950-963.
- Levine, R. (1992) "Financial Intermediary Services and Growth" *Journal of Japanese and International Economics* vol.6 pp.383-405.
- Li, H., L. Squire, and H. Zou (1998) "Explaining International and Intertemporal Variations in Income Inequality" *Economic Journal* vol.108 pp.26-43.
- Loury, G. C. (1981) "Intergenerational Transfers and the Distribution of Earnings" *Econometrica* vol.49 pp.843-867.
- Ljungqvist, L. (1993) "Economic Underdevelopment: The Case of a Missing Market for Human Capital" *Journal of Development Economics* vol.40 pp.219-239.
- Ljungqvist, L. and T. J. Sargent *Recursive Macroeconomic Theory* edited by M.I.T. Press, Cambridge MA (2004)
- Lucas, R. E. (1988) "On the Mechanics of Economic Development" *Journal of Monetary Economics* 22 pp.3-42.
- Lucas, R. E. (1990) "Supply-side Economics: An Analytical Review" *Oxford Economic Papers* vol.42 pp.293-316.
- McKinnon, R. I. (1973) *Money and Capital in Economic Development*. Washington: Brookings Institute.
- Manuelli, R. (1986) "Topics in Intertemporal Economics", University of Minnesota Ph.D. Dissertation.
- Mino, K. (1996) "Analysis of a Two-Sector Model of Endogenous Growth with Capital Income Taxation" *International Economic Review* 37 pp.227-251.
- Pagano, M. and T. Jappelli (1993) "Information Sharing in Credit Markets" *Journal of Finance* vol.43 pp.1693-1718.

- Palivos, T. and C. K. Yip (2007) “Illegal Immigration in a Heterogeneous Society” Discussion Paper Series WP 2007-02, Department of Economics, University of Macedonia Greece.
- Papanek, G. F. and O. Kyn (1986) “The Effect on Income Distribution of Development, the Growth Rate and Economic Strategy” *Journal of Development Economics* vol.23 pp.55-65.
- Paukert, F. (1973) “Income Distribution at different levels of development: A survey of evidence” *International Labor Review* vol.108.
- Ray, D. (1998) *Development Economics*. Princeton University Press.
- Rebelo (1991) “Long-Run Policy Analysis and Long-Run Growth” *Journal of Political Economy* 99 pp.500-521.
- Saint-Paul, G. (1992) “Technological Choice, Financial Markets and Economic Development” *European Economic Review* vol.36 pp.763-781.
- Samuelson, P. A. (1968) “The Two-Part Golden Rule Deduced as the Asymptotic Turnpike of Catenary Motions” *Western Economic Journal* vol.6, pp.85-89.
- Sapienza, P. (2002) “The Effects of Banking Mergers on Loan Contracts” *Journal of Finance* vol.57 pp.329-368.
- Schechtman, J., and V. Escudero (1977) “Some Results on an ‘Income Fluctuation Problem’ ” *Journal of Economic Theory* vol.16 pp.151-166.
- Shaw, E. S. (1973) *Financial Deepening in Economic Development*. New York: Oxford University Press.
- Snow, A. and R. S. Warren, Jr. (1990) “Human Capital Investment and Labor Supply under Uncertainty” *International Economic Review* vol.31 pp.195-206.
- Stiglitz, J. E. and A. Weiss (1981) “Credit Rationing in Markets with Imperfect Information” *American Economic Review* vol.71 pp.393-410.
- Stiglitz, J. E. and A. Weiss (1987) “Macroeconomic Equilibrium and Credit Rationing” Working Paper No.2164, National Bureau of Economic Research.
- Stokey, N. L. (1991) “Human Capital, Product Quality, and Growth” *The Quarterly Journal of Economics* vol. 106 pp. 587-616.
- Tamura, R. (1991) “Income Convergence in an Endogenous Growth Model” *Journal of Political Economy* vol.99 pp.522-540.

- Tsiddon, D. (1992) “A Moral Hazard Trap to Growth” *International Economic Review* vol.33 pp.299-321.
- Zeira, J. (1991) “Credit Rationing in an Open Economy” *International Economic Review* vol.32 pp.959-972.